

Math 215 —Sample Exam 2

Spring 2009

All problems are weighted equally. Partial credit will be given only if your answer makes sense. You must give reasons for your answers.

1. (a) If X, Y are sets of m and n elements respectively, state how many (i) functions (ii) injective functions there are from X to Y .
(b) Find the number of permutations (bijective functions) $X \rightarrow X$ where $X = \{1, 2, 3, 4\}$ such that $f(i) \neq i$ for any i . [Hint: Find those functions for which $f(i) = i$ for some i , and subtract from the total number. See the problem 17 on p.185 of the text.]
2. (a) State the Binomial Theorem.
(b) Show that $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$.
3. (a) Define the greatest common divisor d of two non-zero integers a and b .
(b) Let a, b be non-zero even integers such that $\gcd(a, b) = 2$. Show that $\gcd(a/2, b/2) = 1$.
4. A committee of 7 is to be formed from a group of 10 people.
(a) At least one of two individuals A and B is to be on the committee. Let X be the set of those committees of 7 which include A and let Y be the set of those committees of 7 which include B. Find the number of committees which include either A or B (perhaps both). Express the set of these committees in terms of X and Y.
(b) Suppose any committee member can fill more than one slot on the committee. (For example, assume 7 areas are represented and all 10 people are experts in the 7 areas.) How many committees can be formed?
- 5.(a) Using congruences, show that an integer of the form $521 \cdot 12^k + 1$, $k \geq 1$, k odd is divisible by 13.
(b) Find the remainders mod 7, 11 and 13 of 1111118111111. State the theorem you are using here.