Homework 10

MCS 421 Combinatorics

Problem 8.1. *Solution.* See end of Chapter 7.

Problem 8.2. *Solution.* Let M_n denote the set of $2 \times n$ arrays that satisfy the conditions. Let let P_n denote the number of paths from $(0,0)$ to (n,n) that lies on or below the $y = x$ lines. Hence $|P_n| = C_n = (1/(n+1))\binom{2n}{n}$. Each path $p \in P_n$, we will think of as a sequence of U's and R's of length $2n$, such that there are n U's and n R's, and at any point, there are at least as many R's than U's in the sequence (from left to right). The number of such paths is $|P_n|$. We now represent this path by an array in M_n, where the top row contains to the step numbers (in order) in which we make R's, and the bottom row lists the step numbers (in order) in which we made a U. Hence P_n corresponds to M_n and therefore $|M_n| = C_n$.

Problem 8.6. The diagonal entries are $3, 1, 4, 0, 0, 0, \ldots$. Hence $h_n = 3\binom{n}{0} + 1\binom{n}{2} + 4\binom{n}{2}$. Therefore

$$\sum_{k=0}^{n} h_k = 3\binom{n+1}{1} + 1\binom{n+1}{2} + 4\binom{n+1}{3}.$$

Problem 8.7. The diagonal from the difference table is $1, -2, 6, -3, 0, 0, \ldots\ldots$ Therefore

$$h_n = \binom{n}{0} - 2\binom{n}{1} + 6\binom{n}{2} - 3\binom{n}{3}.$$

Therefore

$$\sum_{k=0}^{n} h_k = \binom{n+1}{1} - 2\binom{n+1}{2} + 6\binom{n+1}{3} - 3\binom{n+1}{4}.$$