Homework 11

MCS 421 Combinatorics

Problem 8.8. *Solution.* The zero column is 0,1,30,150,240,120,0,0,.... Hence

$$n^2 = \binom{n}{1} + 30\binom{n}{2} + 150\binom{n}{3} + 240\binom{n}{4} + 120\binom{n}{5}.$$

Therefore

$$\sum_{k=0}^{n} k^5 = \binom{n+1}{2} + 30\binom{n+1}{3} + 150\binom{n+1}{4} + 240\binom{n+1}{5} + 120\binom{n+1}{6}.$$

Problem 8.12. *Solution.* Proof by induction and using the recursive formula $S(n,k) = S(n-1,k-1) + kS(n-1,k)$.

Problem 8.14. *Solution.* We have

$$n^p = \sum_{t=0}^{p} S(p,t)[n]_t = \sum_{t=0}^{p} S(p,t)t!\binom{n}{t}.$$

Therefore

$$\sum_{k=0}^{n} k^p = \sum_{t=0}^{p} S(p,t)t!\binom{n+1}{t+1}.$$

Problem 8.15. *Solution.* $k!S(n,k)$ is the number of partitions of $X = \{1,2,...,n\}$ to k distinct boxes such that each box is non-empty. k^n is the number of partitions of X into k distinct boxes. Hence there are $\binom{k}{t}$ ways to select t boxes to be non-empty (the rest are empty), and therefore

$$k^n = \sum_{t=0}^{k} \binom{k}{t} t!S(n,t).$$