

Homework 11

MCS 421 Combinatorics

Problem 8.19. *Solution.* Just like last homework, apply induction on n and use the recurrence $s(n, k) = s(n - 1, k - 1) + (n - 1)s(n - 1, k)$.

Problem 8.21. *Solution.* Easy.

Problem 8.22. *Solution.* Partitions for 6 are 6, 51, 42, 411, 33, 321, 3111, 222, 2211, 21111, 111111. $p_6 = 11$. Partition for 7 is 7, 61, 52, 511, 43, 421, 4111, 331, 322, 3211, 31111, 2221, 22111, 211111, 1111111. $p_7 = 15$.

Problem 8.23. *Solution.* Maximal partition is n . Minimal is $1 + 1 + \cdots + 1 = n$.

Problem 8.26. *Solution.* $12 = 4 + 3 + 2 + 2 + 1$. $15 = 5 + 3 + 3 + 2 + 1 + 1$. $20 = 4 + 4 + 4 + 4 + 2 + 2$. $21 = 6 + 5 + 4 + 3 + 2 + 1$. $29 = 6 + 6 + 5 + 4 + 3 + 3 + 1 + 1$.

Problem 8.27. *Solution.* For n odd, place $(n + 1)/2$ points along the 1st column and 1st row. For n even, place $n/2$ points in the first row and first column (this is only $n - 1$ dots since the the top left is repeated), then put the last dot in the “corner”.