Homework 3

Due Monday, April 20

Problem 1. Prove that if G is an interval graph, then $\chi(G) \leq \omega(G)$.

Problem 2. Let $\mathcal{G}$ be the class of intersection graphs of axis-parallel rectangles in the plane. Prove that $\mathcal{G}$ is χ-bounded.

Problem 3. Let $\mathcal{F}$ be a system of finitely many closed intervals on the real line. Prove that $\nu(\mathcal{F}) = \tau(\mathcal{F})$.

Problem 4. For a graph G, let $\mathcal{F} = \{N(v) : v \in V(G)\}$ be the set system of vertex neighborhoods, where $N(v) = \{u : uv \in E(G)\}$.

(a) Prove that there is a constant d_0 such that the VC-dimension of $\mathcal{F}$ is at most d_0 for all planar graphs G.

(b) Show that for every C there is a $d = d(C)$ such that if G is a graph in which every subgraph on n vertices has at most Cn edges, for all $n \geq 1$, then the VC-dimension of $\mathcal{F}$ is at most d.

Problem 5. Let $X \subset \mathbb{R}^2$ be a $(4k+1)$-point set, and let $\mathcal{F} = \{conv(Y) : Y \subset X, |Y| = 2k+1\}$.

(a) Verify that $\mathcal{F}$ has the (4,3)-property, that is, every 4 members in $\mathcal{F}$ contains three members with a nonempty intersection. Also show that if X is in convex position, then $\tau(\mathcal{F}) \geq 3$.

(b) Show that $\tau(\mathcal{F}) \leq 5$.