Homework 3

MCS 421 Combinatorics

Problem 2.39 (a) $\binom{20}{6}$. (b) Label the sticks 1,..., 20. Let $x_1 < \cdots < x_6$ be a valid choice. Then define, $y_1 = x_1 - 1$, $y_2 = x_2 - x_1 - 2$, ..., $y_6 = x_6 - x_5 - 2$ and $y_7 = 20 - x_6$. Notice that a solution $x_1, ..., x_6$ correspond to an integral solution for $y_1, ..., y_7$ such that $y_i \geq 0$ and $y_1 + \cdots + y_7 = 9$. Hence the number of solutions is $\binom{15}{6}$. (c) As before, let $y_1 = x_1 - 1$, $y_2 = x_2 - x_1 - 3$, ..., $y_6 = x_6 - x_5 - 3$ and $y_7 = 20 - x_6$. Then we find solutions y_i such that $y_i \geq 0$ and $y_1 + \cdots + y_7 = 4$. Hence the answer is $\binom{10}{6}$.

Problem 2.60 Total number of ways to select 15 bagels is solving the integral solutions $x_1, ..., x_6$, $x_i \geq 0$ and $x_1 + \cdots + x_6 = 15$. Number of ways to select 15 bagels so that you get at least one of each kind is solving the integral solutions of $y_i \geq 0$, and $y_1 + \cdots + y_6 = 9$. Hence the probability of getting at least one bagel of each kind is $\frac{\binom{14}{5}}{\binom{20}{5}}$.

For the second solution, we solve $z_i \geq 0$ and $z_1 \cdots z_6 = 12$. Hence the probability is $\frac{\binom{17}{5}}{\binom{20}{5}}$.

Problem 3.4-5-6 Given integers $n \geq 1$ and $k \geq 2$ suppose that $n + 1$ distinct elements are chosen from $\{1, 2, ..., kn\}$. We show that there exist two that differ by less than k. Partition $\{1, 2, ..., kn\} = \cup_{i=1}^{n} S_i$ where $S_i = \{ki, ki - 1, ki - 2, ki - k + 1\}$. Among our $n + 1$ chosen elements, there exist two in the same S_i. These two differ by less than k.

Problem 3.18 Divide the square into four 1×1 squares. By pigeonhole, there is a 1×1 square that contains at least 2 points, and hence has distance at most $\sqrt{2}$.