Practice Test 2, Math 310

Problem 1. Find an invertible matrix P and a matrix C of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ such that $A = PCP^{-1}$, where $A = \begin{pmatrix} 5 & -5 \\ 1 & 1 \end{pmatrix}$.

Problem 2. Let $A = \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$. Compute A^{100}.

Problem 3. Find the eigenvalues of the matrix $\begin{pmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.

Problem 4. Let $b_1 = \begin{pmatrix} -1 \\ 8 \end{pmatrix}, b_2 = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$ and $c_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, c_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Find the change of coordinate matrix from $B = \{b_1, b_2\}$ to $C = \{c_1, c_2\}$.

Problem 5. Find the bases for $Col(A), Row(A), Nul(A)$, where $A = \begin{pmatrix} -3 & 6 & -1 & 1 \\ 1 & -2 & 2 & 3 \\ 2 & -4 & 5 & 8 \end{pmatrix}$.

Problem 6. Let P be the vector space of all polynomials of degree at most 3, i.e., $p(t) = a + bt + ct^2 + dt^3$. Let H be a subset of P, which consists of the polynomials in P with the property that $p(0) = 0$. Is H a subspace? Prove or disprove you answer.