

Math 310, Quiz 2 solutions

Problem 1. Solve the matrix equation $\begin{pmatrix} 1 & 5 & -2 & 0 \\ -3 & 1 & 9 & 5 \\ 4 & -8 & -1 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -7 \\ 9 \\ 0 \end{pmatrix}$.

Solution. After performing elementary row operations, we have the augmented matrix $\begin{pmatrix} 1 & 5 & -2 & 0 & -7 \\ 0 & 16 & 3 & 5 & -12 \\ 0 & 0 & 7 & 9 & 4 \end{pmatrix}$. Back solving gives $x_4 = \text{free variable}$, $x_3 = (4 - 9x_4)/7$,

$$x_2 = (-12 - 3((4 - 9x_4)/7) - 5x_4)/16,$$

and

$$x_1 = -7 - 5((-12 - 3((4 - 9x_4)/7) - 5x_4)/16) - 2((4 - 9x_4)/7).$$

Problem 2. Determine if the vector $\begin{pmatrix} 11 \\ -5 \\ 9 \end{pmatrix}$ is a linear combination of the vectors

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} -6 \\ 7 \\ 5 \end{pmatrix}.$$

Solution. This corresponds to the augmented matrix $\begin{pmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{pmatrix}$. After per-

forming elementary row operations, we have $\begin{pmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{pmatrix}$. We have a pivot in each

column of the coefficient matrix, and therefore the vector $\begin{pmatrix} 11 \\ -5 \\ 9 \end{pmatrix}$ is a linear combination

of the vectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} -6 \\ 7 \\ 5 \end{pmatrix}$.