Math 310, Quiz 3 solutions

Problem 1. Determine if the set of vectors $\left\{ \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ -8 \end{pmatrix}, \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} \right\}$ is independent or dependent. Justify your answer.

Solution. The augmented matrix of the homogeneous system is $\begin{pmatrix} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{pmatrix}$. After performing elementary row operations, we have the echelon form matrix $\begin{pmatrix} 2 & -8 & 1 & 0 \\ 0 & 5 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{pmatrix}$. This matrix has a pivot in every column, and therefore the vectors are independent.

Problem 2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ to $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ to $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$. What is $T(v)$, where $v = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

Solution. Since T is linear, we have $T(v) = 2\begin{pmatrix} 2 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 25 \\ 8 \end{pmatrix}$.