A related problem, called an annuity, is to find the present value $P(0)$ of a series of $N$ yearly fixed payments of $c$ (on the last day of the year). Here $r$ is the interest rate and the compounding is continuous. The pricing problem is a FVP of the form

\[
\frac{dP}{dt} = rP - c \sum_{j=1}^{N-1} \delta(t - j), \quad t < N, \text{ with } P(N) = c. \tag{1}
\]

We solve (1) by first re-arranging the terms and then multiplying by an integrating factor $e^{-rt}$ to obtain

\[
e^{-rt}\left\{\frac{dP}{dt} - rP\right\} = -ce^{-rt} \sum_{j=1}^{N-1} \delta(t - j).
\]

We recognize that the left side of the equation is the derivative of a product so that we have

\[
\frac{d}{dt}[e^{-rt}P] = -ce^{-rt} \sum_{j=1}^{N-1} \delta(t - j)
\]

We are now able to integrate both sides of the equation from 0 to $N$ with respect to $t$ to obtain

\[
e^{-rN}P(N) - P(0) = -c \int_{0}^{N} e^{-rt} \sum_{j=1}^{N-1} \delta(t - j)dt.
\]

Using the properties of the $\delta$ function, the final condition $P(N) = c$, and solving for $P(0)$ we obtain

\[
P(0) = c \sum_{j=1}^{N-1} e^{-rj} + ce^{-rN} = c \sum_{j=1}^{N} e^{-rj}.
\]

This is the standard formula for the present value of this annuity.