Math 586 Computational Financial

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Homework 1 - Compound Interest and Payoff Diagrams

1. What is the *future value* of a \$1000 investment after 5 years with continuous compounding assuming an interest rate of 5% p.a. for the first 6 months, 5 1/4% p.a. for the next year, and 5% p.a. for the remaining time.

2. What is the present value of investment that with payments of \$1000 for 5 consecutive years starting 10 years from today. Assume an interest rate of 7% with continuous compounding.

3. Which of the following sequence of yearly payments (starting one year from now) is preferable, i.e. the largest present value, if $r = 10\%$ p.a. continuously compounded.

   (a) 1200, 1400, 1600, 1800, 2000

   (b) 1600, 1600, 1500, 1500, 1500

   (c) 2000, 1600, 1400, 1200, 1000

   Repeat if $r = 20\%$, and 30%.

4. Consider a coupon bond maturing to \$1 in one year. Every six months the bond pays a coupon of c. The interest rate $r(t)$ varies during the year and is assumed known.

   (a) If $V(t)$ is the value of the bond at time t, give the pricing equation for $V(t)$.

   (b) Derive a formula for $V(t)$. If r is constant, show that this formula reduces to the standard result.

Payoff Diagrams in Matlab: Option strategies can be illustrated graphically in Matlab. It is helpful to use a standard notation. First, we define the value of a European call option as $c(S, E, t)$ where

- S is the price of the underlying stock at time t, today
- E is the exercise or strike price
- T is expiration date

Similarly the value of a European put option is denoted by $p(S, E, t)$. To distinguish between European and American style options, the prices of the American options will be in upper case, i.e. C and P. Some texts use the notation $c(S, t)$ omitting the exercise price while other presentations use the notation $c(S, E, T - t)$. Here $T - t$ is the time till expiration from the current time t. There are other representations.

At expiration, the value (or payoff) of an option is independent of the original cost or premium. For example, we use the formulas:

$$c(S, E, T) = \max(0, S - E) \tag{1}$$

and

$$p(S, E, T) = \max(0, E - S). \tag{2}$$

Using the above notation, the value of a short call position at expiration is given by

$$-c(S, E, T) = -\max(0, S - E).$$

The value of a combination of options can be written in terms of the values of the individual options.

Write a *Matlab* program, based on the program below, to solve the following problems. For the problems, we assume that the current price of Yahoo (YHOO) is 80.

5. Plot the payoff diagram for a short option position at expiration consisting of selling 1 put of YHOO with $E = 75$ for 4. Thus, you need to plot $-p(S, 75, T)$ as a function of S.

6. **Straddle:**We buy a call for 3 with exercise price of 85 and buy a put for 7 with exercise price of 85. Both the put and call have the same exercise date. Give a formula for the value of the straddle at T using the above notation. Plot the payoff diagram and the profit and loss (P & L) diagram.

7. **Bull Spread:** The position consists of buying a call option for 7 with exercise price 75 and selling a call option with exercise price 85 for 3. Give a formula for the value of the bull spread at T. Plot the payoff diagram.

8. **Long Condor:** The position, also called an elongated butterfly, consists of

	E	Option premium
Long 1 call	70	10
Short 1 call	75	7
Short 1 call	80	4
Long 1 call	85	2

   All options have the same expiration date T. Give a formula for the value of the position at T. Plot the payoff diagram at expiration.

9. A range forward contract is a variation of the standard forward contract in which at the delivery date T the holder pays E_1 for the asset if the asset price $S_T < E_1$, pays E_2 if $S_T > E_2$, and pays the spot price at T if $E_1 < S_T < E_2$, where $E_1 < E_2$.

   (a) Use European options to replicate the range forward payoff.

   (b) Graph the value or payoff using Matlab.

   (c) What is the advantage of the range forward over the standard forward contract? Should the holder have to pay to enter the contract?

```
%Higham --  Chapter 1
%
% Plots of payoff diagram

clf
E1 = 2;
E2 = 4;

S = linspace(0,6,100);
B = max(S-E1,0)-max(S-E2,0);
plot(S,B)
ylim([0,3])

xlabel('S') ylabel('B') title('Bull Spread Payoff') grid on
```