Math 586 - Computational Financial - C. Tier - Spring 2006
Homework - 2

1. Consider a European call option on a stock with current price $S_t = \$15$. The option expires in 6 months (at T) and has strike price \$15. At expiration, we assume the stock price (S_T) is either 14 w.p. 0.4 or 18 w.p. 0.6. The risk-free interest rate is $r = 5\%$. Let c_t be the price of the option today and c_T the value at expiration. The value at expiration is the payoff function.

   (a) Using the real world probabilities, compute a price of the option using the discounted value of the expected payoff, i.e. $\hat{c}_t = e^{-r(T-t)}\mathrm{E}[\text{payoff}]$, where the expectation is with respect to the objective or real-world probabilities.

   (b) Using the risk-neutral approach, compute the risk-neutral probabilities and the *fair* price of the option, $c_t = e^{-r(T-t)}\mathrm{E}^Q[c_T]$.

   (c) Repeat the pricing using the replication approach.

   (d) If an investor incorrectly prices the option using (a). Describe a trade that you can use with this investor to obtain a certain profit.

2. Price the same option using the data described in 1. except that it is now 3 months till expiration, i.e. $T - t = 1/4$ yr. Use the risk-free portfolio approach. Compare with the price in 1.

3. Suppose the option in 1. is a European put option. Compute its price using the data in 1.

4. Consider a forward contract on a non-dividend-paying stock that matures in 3 months. Suppose the delivery price is \$42, the current price of the stock is \$40, and the 3-month risk-free rate of interest it 5% p.a. (assume continuous compounding). Is this an arbitrage opportunity and if so, describe the position an arbitrageur would take and his risk-free profit.

5. A 1-year long forward contract on a non-dividend-paying stock is entered into when the stock price is \$40 and risk-free interest rate is 10% p.a. with continuous compounding.

   (a) What is the forward price or delivery price of the contract?

   (b) Six months later the price of the stock is \$45 and the risk-free interest rate is till 10% p.a. What is the forward price of of a new forward contract with the same delivery date?

   (c) What is the value of the original forward contract in six months if the stock price is \$45?