1. Consider a European put option $p$ as a function of stock price $S$ whose value is determined by using the Black-Scholes formula. Assume the parameter values: $E = 50$, $r = 0.05$, and $\sigma = 0.20$. Please make a graph (on the same page) for each of the following time to expiration $T - t = 1.0$, $0.3846$, $0.1$. Also, include the graph of the payoff function. Use your previous programs including from the Notes on Hedging.

2. Derive the formula for the value, $bc(S, t)$, of a binary call where $bc(S, T) = H(S - E)$. Here $H(x)$ is the Heaviside step function. You need not solve the Black-Scholes equation directly. Instead use the risk-neutral approach. Also, compute and graph $\Delta$ if $r = 0.10$, $\sigma = 0.20$, and $E = 10$ for $T - t = 1.0$, $T - t = 0.5$, and $T - t = 0.1$. How does the hedge ratio behave as $t \rightarrow T$.

3. Consider the following option data for Time Warner Inc. (TWX) traded on the NYSE. The options expire in April 2005 but have different strike prices. Assume the the current price of TWX is $S = 18.67$, the time to expiration is $T - t = 1/12$, and the risk-free rate is $r = 2.70\%$. Modify and use the Higham’s Matlab programs (make sure you understand them!) below to compute the implied volatility for each option. Include both a table and a graph of the implied vol. versus the strike price.

<table>
<thead>
<tr>
<th>Strike</th>
<th>Symbol</th>
<th>Last</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.00</td>
<td>TWXPQ</td>
<td>2.74</td>
<td>1</td>
</tr>
<tr>
<td>17.00</td>
<td>TWXDR</td>
<td>1.80</td>
<td>13</td>
</tr>
<tr>
<td>18.00</td>
<td>TWXDS</td>
<td>1.00</td>
<td>54</td>
</tr>
<tr>
<td>19.00</td>
<td>TWXDT</td>
<td>0.40</td>
<td>722</td>
</tr>
<tr>
<td>20.00</td>
<td>TWXDD</td>
<td>0.20</td>
<td>200</td>
</tr>
</tbody>
</table>

4. Consider a European call option with strike $E$ and expiry $T$ on a stock that pays a discrete cash dividend $D$ at $t_D < T$. The price of stock today (time $t$) follows the standard geometric Brownian motion and $r$ is the risk-free rate.

(a) Formulate a Black-Scholes type problem for the price of the call option $c(S, t)$ for times $t < t_D$. Explicitly state the appropriate final condition.

(b) A standard formula used for the solution for $c$ in (a) is simply replace $S \rightarrow S - D e^{-r(t_D-t)}$ in the standard Black-Scholes formula. Verify whether or not this is the solution of the problem you stated in (a). Does it satisfy the final condition and the condition at $t_D$.

(c) What difficulties are there in implementing the solution in (b)? You might suggest some data and try to graph the result.

%CH14    Higham’s Program for Chapter 14
%
% Computes implied volatility for a European call - calls program
% from chapter 10
********** parameters **********
\[ r = 0.03; \quad S = 2; \quad E = 2; \quad T = 3; \quad \tau = T; \quad \sigma_{\text{true}} = 0.3; \]
[C_{\text{true}}, \quad C_{\delta}, \quad P, \quad P_{\delta}] = \text{ch08}(S,E,r,\sigma_{\text{true}},\tau);

%starting value
\[ \hat{\sigma} = \sqrt{2 \times \frac{\text{abs}(\log(S/E) + r \times T)}{T}}; \]

************************** Newton's Method **************************
tol = 1e-8;
sigma = \hat{\sigma};
sigmat = 1;
k = 1;
kmax = 100;
while (sigmadiff >= tol & k < kmax)
    \[ [C, \quad C_{\delta}, \quad C_{\vega}, \quad P, \quad P_{\delta}, \quad P_{\vega}] = \text{ch10}(S,E,r,\sigma,\tau); \]
    increment = \frac{C - C_{\text{true}}}{C_{\vega}};
    sigma = sigma - increment;
    k = k+1;
    sigmadiff = abs(increment);
end
sigma

function \[ [C, \quad C_{\delta}, \quad C_{\vega}, \quad P, \quad P_{\delta}, \quad P_{\vega}] = \text{ch10}(S,E,r,\sigma,\tau) \]
% Higham's program for Chapter 10
% This is a MATLAB function
%
% Input arguments: S = asset price at time t
% E = Exercise price
% r = interest rate
% sigma = volatility
% tau = time to expiry (T-t)
%
% Output arguments: C = call value, C_{\delta} = delta value of call
% C_{\vega} = vega value of call
% P = Put value, P_{\delta} = delta value of put
% P_{\vega} = vega value of put
%
% function \[ [C, \quad C_{\delta}, \quad C_{\vega}, \quad P, \quad P_{\delta}, \quad P_{\vega}] = \text{ch10}(S,E,r,\sigma,\tau) \]
if \tau > 0
    \[ d1 = (\log(S/E) + (r + 0.5 \times \sigma^2) \times (\tau))/(\sigma \times \sqrt{\tau}); \]
    \[ d2 = d1 - \sigma \times \sqrt{\tau}; \]
    \[ N1 = 0.5 \times (1 + \text{erf}(d1/\sqrt{2})); \]

\begin{verbatim}
N2 = 0.5*(1+erf(d2/sqrt(2)));
C = S*N1-E*exp(-r*(tau))*N2;
Cdelta = N1;
Cvega = S*sqrt(tau)*exp(-0.5*d1^2)/sqrt(2*pi);
P = C + E*exp(-r*tau) - S;
Pdelta = Cdelta - 1;
Pvega = Cvega;
else
C = max(S-E,0);
Cdelta = 0.5*(sign(S-E) + 1);
Cvega = 0;
P = max(E-S,0);
Pdelta = Cdelta - 1;
Pvega = 0;
end

5. Consider a perpetual American call option on a stock that pays a continuous yield \( q \) and strike \( E \). Let \( \tilde{c}(S) \) denote the price of the option and let \( S^* \) be the optimal exercise price. Assume the stock price follows the SDE: \( ds = \mu S dt + \sigma S dW \).

(a) State the complete problem for \( \tilde{c}(S) \) and \( S^* \). Explain any extra conditions.
(b) Derive formulas for \( \tilde{c}(S) \) and \( S^* \).
(c) Assume the data: \( r = 5\% \), \( q = 1\% \), \( E = 20 \), and \( \sigma = 20\% \). Compute \( S^* \) and graph \( \tilde{c}(S) \) on the domain \( S \in [0,30] \).

6. There are several relationships between the prices of put and calls. We have already derived the put-call parity for European options on assets with or without dividends. In general, put-call parity is not valid for American options. Another relationship is symmetry, which is valid for European, American, and Perpetual American options. Let \( \tilde{c}(S; E, r, q) \) be the price (extended notation) of a perpetual American call option on an asset that pays a continuous yield \( q \) (see the previous problem).

(a) Using problem for \( \tilde{c} \) in the previous exercise, derive the put-call symmetry

\[ \tilde{c}(S; E, r, q) = E S \tilde{p}(1/S; 1/E, q, r). \]

Here \( \tilde{p}(S) \) is the price of a corresponding perpetual American put.
(b) Compute \( \tilde{p}(18; 20, 0.05, 0.01) \) using put-call symmetry. What is the corresponding value of \( S^* \)?
\end{verbatim}