Computational Finance - Spring 2007 - C. Tier
Homework - 5

1. Consider a European put option p as a function of stock price S whose value is determined by using the Black-Scholes formula. Assume the parameter values: $E = 50$, $r = 0.05$, and $\sigma = 0.20$, Please make a graph (on the same page) for each of the following time to expiration $T - t = 1.0,\ 0.3846,\ 0.1$. Also, include the graph of the payoff function. Use your previous programs including from the *Notes on Hedging*.

2. Derive the formula for the value, $bc(S, t)$, of a binary call where $bc(S, T) = H(S - E)$. Here $H(x)$ is the Heaviside step function. You need not solve the Black-Scholes equation directly. Instead use the risk-neutral approach. Also, compute and graph Δ if $r = 0.10$, $\sigma = 0.20$, and $E = 10$ for $T - t = 1.0$, $T - t = 0.5$, and $T - t = 0.1$. How does the hedge ratio behave as $t \to T$.

3. Consider the following option data for Time Warner Inc. (TWX) traded on the NYSE. The options expire in April 2005 but have different strike prices. Assume the the current price of TWX is $S = 18.67$, the time to expiration is $T-t = 1/12$, and the risk-free rate is $r = 2.70\%$. Modify and use the Matlab programs below (make sure you understand them!) below to compute the implied volatility for each option. Include both a table and a graph of the implied vol. versus the strike price.

Strike	Symbol	Last	Volume
16.00	TWXPQ	2.740	1
17.00	TWXDR	1.800	13
18.00	TWXDS	1.000	54
19.00	TWXDT	0.400	722
20.00	TWXDD	0.200	200

4. Consider the prices of two assets S_1 and S_2 whose dynamics follow the SDE's:

$$\frac{dS_1}{S_1} = \mu_1 dt + \sigma_1 dW_1$$
$$\frac{dS_2}{S_2} = \mu_2 dt + \rho\sigma_2 dW_1 + \sqrt{1-\rho^2}\sigma_2 dW_2.$$

   Here $W_1(t)$ and $W_2(t)$ are uncorrelated standard Brownian motions.

   (a) Verify that instantaneous returns are correlated, i.e. Compute E$[\frac{dS_1}{S_1}\frac{dS_2}{S_2}]$.

   (b) If $Y = f(S_1, S_2) = S_1 S_2$ then find the SDE for Y.

5. Consider the static hedge example on page 56 of the text for a derivative with an exponential payoff $\phi(S) = e^{S-X} - 1$, where X is the strike of the exponential option.

   (a) Using (1.245) with $S_0 = X$ and $S_1 = \infty$, state precisely the replication formula for $\phi(S)$ in terms of bonds, assets and vanilla options. What the are the weights of each instrument in the hedge?

   (b) Analytically evaluate the integral involving the vanilla call options to verify that the value of the hedge portfolio is indeed equal to the payoff of the exponential option.

   (c) Consider the discretized version of the static hedge given by (1.250) using only two options with strikes $K_1 = 10.75$ and $K_2 = 12.25$. Assume $X = 10$ on the interval [10,13]. Graph the exponential payoff and the value of the discrete hedge portfolio (on the same graph but with different line types) from 10 to 13. Does the hedge look accurate?

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%CH14    Higham's Program for Chapter 14
% Computes implied volatility for a European call - calls program
%%%%%%%%% parameters %%%%%%%%%%%%%%%%
r = 0.03; S = 2; E = 2; T = 3; tau = T; sigma_true = 0.3;
[C_true, Cdelta, P, Pdelta] = ch08(S,E,r,sigma_true,tau);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%starting value
sigmahat = sqrt(2*abs( (log(S/E) + r*T)/T ) );
%%%%%%%%%%%%%% Newton's Method %%%%%%%%
tol = 1e-8;
sigma = sigmahat;
sigmadiff = 1;
k = 1;
kmax = 100;
while (sigmadiff >= tol & k < kmax)
    [C, Cdelta, Cvega, P, Pdelta, Pvega] = ch10(S,E,r,sigma,tau);
    increment = (C-C_true)/Cvega;
    sigma = sigma - increment;
    k = k+1;
    sigmadiff = abs(increment);
end
sigma

function [C, Cdelta, Cvega, P, Pdelta, Pvega] = ch10(S,E,r,sigma,tau)
% Higham's program for Chapter 10
% This is a MATLAB function
%
% Input arguments: S = asset price at time t
%                  E = Exercise price
%                  r = interest rate
%                  sigma = volatility
%                  tau = time to expiry (T-t)
%
% Output arguments: C = call value, Cdelta = delta value of call
%                   Cvega = vega value of call
%                   P = Put value, Pdelta = delta value of put
%                   Pvega = vega value of put
%
%   function [C, Cdelta, Cvega, P, Pdelta, Pvega] = ch10(S,E,r,sigma,tau)

if tau > 0
   d1 = (log(S/E) + (r + 0.5*sigma^2)*(tau))/(sigma*sqrt(tau));
   d2 = d1 - sigma*sqrt(tau);
   N1 = 0.5*(1+erf(d1/sqrt(2)));
   N2 = 0.5*(1+erf(d2/sqrt(2)));
   C = S*N1-E*exp(-r*(tau))*N2;
   Cdelta = N1;
   Cvega = S*sqrt(tau)*exp(-0.5*d1^2)/sqrt(2*pi);
   P = C + E*exp(-r*tau) - S;
   Pdelta = Cdelta - 1;
   Pvega = Cvega;
else
   C = max(S-E,0);
   Cdelta =  0.5*(sign(S-E) + 1);
   Cvega = 0;
   P = max(E-S,0);
   Pdelta = Cdelta - 1;
   Pvega = 0;
end
```