Computational Finance - Spring 2007 - Homework - 6
Charles Tier

1. Consider a perpetual American call option on a stock index that pays a continuous yield q and strike E. Let $\tilde{c}(S)$ denote the price of the option and let S^* be the optimal exercise price. Assume the stock index price follows the SDE: $ds = \mu S dt + \sigma S dW$.

   (a) State the complete problem for $\tilde{c}(S)$ and S^*. Explain any extra conditions.

   (b) Derive formulas for $\tilde{c}(S)$ and S^*.

   (c) Give the replicating portfolio for this option. State precisely the hedge ratio and the cash position.

   (d) Assume the data: $r = 5\%$, $q = 1\%$, $E = 20$, and $\sigma = 20\%$. Compute S^* and graph $\tilde{c}(S)$ on the domain $S \in [0, 30]$.

2. There are several relationships between the prices of put and calls. We have already derived the put-call parity for European options on assets with or without dividends. In general put-call parity is not valid for American options. Another relationship is symmetry, which is valid for European, American and perpetual American options. Let $\tilde{c}(S; E, r, q)$ be the price (extended notation) of a perpetual American call option on an asset that pays a continuous yield q (see the previous problem).

   (a) Using problem for $\tilde{c}$ in the previous exercise, derive the put-call symmetry

   $$\tilde{c}(S; E, r, q) = ES\tilde{p}(1/S; 1/E, q, r).$$

   Here $\tilde{p}(S)$ is the price of a corresponding perpetual American put.

   (b) Compute $\tilde{p}(18; 20, 0, 05, 0.01)$ using put-call symmetry. What is the corresponding value of S^*?

3. Consider the European down-and-out call option discussed in class. Assume that the barrier is X and the exercise price $E > X$. The underlying asset pays no dividends and the risk-free rate is a constant r.

   (a) State completely the problem for the price $V(S, t)$ for this option.

   (b) Verify that the formula derived in class satisfies the pricing problem including all conditions.

   (c) Derive the Greeks Δ ,Θ, and Γ for the option. Is the relationship $\Theta = \frac{1}{2}\sigma^2 S^2 \Gamma + r(S\Delta - V)$ valid - where does this come from?

   (d) Plot the solution if $X = 10$, $E = 20$, $r = .05$, and $\sigma = .20$ as a function of initial price S. Choose 3 convenient times that lead to different graphs and include the payoff function. All graphs should be on the same axes and labeled.