MATH 220 **EXAM I** **9.28.01 - high**

noon

Directions: Answer all questions and show all (intermediate) work in the **exam booklet** provided. Start each new question at the **top** of a **new page** and **box** your final answer. Put TI-89 when you use it to compute the solution. Each question is worth 20 points.

1. Solve the IVP:
$$\frac{dy}{dt} = \frac{t}{e^{-y}} - \frac{e^y}{t}$$
   (a) State the name of the method you are using.
   (b) Find a solution.
   (c) Find a solution which satisfies the initial condition $y(1) = 1$.

2. Given the ODE:
$$\frac{dy}{dt} = f(t, y) \text{ where } f(t, y) = -ty + y^2$$
   (a) State Euler's numerical algorithm for this equation using step size $h = 1/10$.
   (b) Given that $y(1) = 1$, compute $y(1)$ and $y(1.1)$ in the case when $f(t, y) = -ty + y^2$.

3. (a) Find the general solution of: $y''(x) + 2y'(x) + y(x) = 0$ and compute the Wronskian of the solution set.
   (b) Find the solution to
$$y''(x) - 4y(x) = -e^x + x \quad y(0) = 0, \;\; y'(0) = 0.$$

4. Given the ODE: $\frac{1}{x}\frac{dy}{dx} + y = x$
   (a) Solve the equation if the initial condition is $y(1) = 1$.
   (b) Make a rough sketch the direction field corresponding to the ODE in your test booklet over the region $0 \le x \le 2, \;\; 0 \le y \le 2$. (Be sure to indicate the direction field on the lines $x = 0, 1, 2$ and $y = 0, 1, 2$.)
   (c) Sketch the solution curve which corresponds to the initial condition $y(1) = 1$ on the direction field sketch from part (b) over the range $0 \le x \le 2$..

5. *Set up* **but do not solve** the problem for $A(t)$ = the amount of salt in the tank (in lbs.) at time t:

   Consider a large tank holding 2,000 gallons of brine solution, initially containing 10 lbs of salt.

   At time $t = 0$, more brine solution begins to flow into the tank at the rate of 2 gal/min. The concentration of salt in the solution entering the tank is $3e^{-t}$ lbs/gal, i.e. varies in time.

The solution inside the tank is well-stirred and is flowing out of the tank at the rate of 5 gal/min.