

Math 310 Exam 1

Name:

October 12, 2001

Answer all 5 questions completely. You must **show all** to receive credit. If you use a calculator, please write *used calculator* where it was used. Try to check your solutions.

1. (a) Find the *row echelon form* (REF) of:  $[A|\mathbf{b}] = \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & -4 \end{array} \right]$  and construct the solution to  $A\mathbf{x} = \mathbf{b}$ .

- (b) Solve the following system using Gaussian elimination on its corresponding augmented matrix.

$$x_1 + 2x_2 + x_4 = 0$$

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$2x_1 + x_2 + x_3 = 0.$$

2. (a) Determine whether the following matrix is singular or nonsingular  $\left[ \begin{array}{ccc} 1 & 2 & 4 \\ 0 & -1 & -3 \\ 2 & 2 & 0 \end{array} \right]$ .

You must explain how you arrived at your conclusion.

- (b) Find the inverse, if possible, of  $A = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{array} \right]$ .

3. (a) Find the determinant of  $A^{-1}$  if  $A = \left[ \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 4 \end{array} \right]$ .

- (b) Use Cramers' rule to solve 
$$\begin{array}{rcl} x_1 + 3x_2 & = & -2 \\ 2x_1 + 5x_2 & = & 1 \end{array}$$

4. Given  $B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 4 & 3 \end{bmatrix}$

(a) Find the rank of  $B$

(b) Find a basis for the null space  $N(B)$ .

(c) Find the a basis for the column space of  $B$ . What is the dimension of the column space?

5. (a) Find the transition matrix to change coordinates in the basis  $\mathbf{v}_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ ,  $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$  to those in the standard basis  $\{\mathbf{e}_1, \mathbf{e}_2\}$ .

(b) Are the vectors  $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$  linearly independent?

(c) Is  $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$  in the span of  $\begin{bmatrix} 4 \\ 0 \\ 5 \end{bmatrix}$  and  $\begin{bmatrix} 6 \\ 0 \\ 1 \end{bmatrix}$  ?