

MCS 471 - Computer Problem 3

This project uses the Matlab function (*invpowerit*) on page 549 that implements the shifted inverse power method for the eigenvalue problem $Ax = \lambda x$. The value of the shift parameter s is an input. Recall how one might guess the values of s or ask in class. For example, if $s = 0$, then this iteration method yields the smallest (in absolute value) eigenvalue and its corresponding eigenvector.

The inputs into the original function *invpowerit* are the matrix A , the value of s , the initial starting vector x , and a maximum number of iterations k . The outputs are the approximations to the smallest eigenvalue and its corresponding eigenvector.

You must introduce the following modifications to *invpowerit*:

1. Instead of using the `\`command to invert the matrix As in the iteration, use Matlab's *LU* decomposition command and then invert the L and U separately using the `\`command. Please do this efficiently!!
2. Add a second stopping criterion: when the 2 norm of the difference between two consecutive eigenvector iterates is less than 10^{-4} .

Call your new function: *myinvpowerit*.

For each of the matrices below, use your modified function *myinvpowerit* to compute eigenvalues and eigenvectors for the given shifts. Choose $k = 1000$. In addition, compare your results with the results obtained using the build-in Matlab command *eig*:

```
[lambda, v] = eig(A)
```

Tabulate your results for each shift and give the relative errors using the Matlab function *eig* as exact. You must also submit the function *invpowerit*.

1. Consider:

$$A = \begin{pmatrix} 2.5 & -2.5 & 3.0 & 0.5 \\ 0.0 & 5.0 & -2.0 & 2.0 \\ -0.5 & -0.5 & 4.0 & 2.5 \\ -2.5 & -2.5 & 5.0 & 3.5 \end{pmatrix}$$

Use the shifts $s = 0, 2.5, 4.5$ and an initial vector with all ones.

2. Consider

$$A = \begin{pmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{pmatrix}$$

Use the shifts $s = 0, 4.2$ and the starting vector $(1, 2, 1)'$. Repeat using the same shifts but with the starting vector $(1, 1, 1)'$. Write a short explanation describing the outcome.