1. Secant: $x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k)$, $k = 1, 2, \ldots$

2. Newton: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$, $k = 0, 1, \ldots$

   or $x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)}$, $k = 0, 1, \ldots$, for $f(x) = (x-r)^m h(x)$, $h(r) \neq 0$.

3. Fixed-Point Iteration: $x_{k+1} = g(x_k)$, $k = 0, 1, \ldots$

4. norms for $\mathbf{x} \in \mathbb{R}^n$: $||\mathbf{x}||_1 = \sum_{i=1}^{n} |x_i| \qquad ||\mathbf{x}||_\infty = \max_{i=1}^{n} |x_i| \qquad ||\mathbf{x}||_2 = \left(\sum_{i=1}^{n} x_i^2 \right)^{1/2}$

5. norms for $A \in \mathbb{R}^{n \times m}$:

$$||A||_1 = \max_{j=1}^{m} \sum_{i=1}^{n} |a_{ij}| \qquad ||A||_\infty = \max_{i=1}^{n} \sum_{j=1}^{m} |a_{ij}|$$

$$||A||_f = \left(\sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij}^2 \right)^{1/2} \qquad ||A|| = \max_{||\mathbf{x}||=1} ||A\mathbf{x}||$$

6. condition number $\text{cond}(A) = ||A||||A^{-1}||$ for $A\mathbf{x} = \mathbf{b}$, $\mathbf{r} = \mathbf{b} - A\bar{\mathbf{x}}$:

$$\frac{||\mathbf{r}||}{||\mathbf{b}||} \frac{1}{||A||||A^{-1}||} \leq \frac{||\mathbf{x} - \bar{\mathbf{x}}||}{||\mathbf{x}||} \leq ||A||||A^{-1}|| \frac{||\mathbf{r}||}{||\mathbf{b}||} \quad \text{or} \quad \frac{||\mathbf{x} - \bar{\mathbf{x}}||}{||\mathbf{x}||} \leq ||A||||A^{-1}|| \frac{||E||}{||A||}, \quad E = A - \bar{A}$$

   for A: $A^T = A$, with $A\mathbf{v}^{(i)} = \lambda_i \mathbf{v}^{(i)}$, and $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_n|$:

$$||A||_2 = |\lambda_1| \qquad ||A^{-1}||_2 = |\lambda_n^{-1}| \qquad ||A||_2 ||A^{-1}||_2 = \left| \frac{\lambda_1}{\lambda_n} \right|$$

7. Newton for system: $f_1(\mathbf{x}) = 0$, $f_2(\mathbf{x}) = 0$, $\ldots$, $f_n(\mathbf{x}) = 0$:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}, \quad k = 0, 1, \ldots, \quad \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \Delta\mathbf{x} = - \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_n(\mathbf{x}) \end{bmatrix}.$$

8. Eigenvalue problem: Find $x \neq 0$, $A\mathbf{x} = \lambda\mathbf{x}$

9. Lagrange interpolation for pts. (x_i, f_i), $i = 0, \ldots n$: $l_i(x) = \prod_{\substack{j=0 \\ \neq i}}^{n} \frac{x - x_j}{x_i - x_j}, \quad p_n(x) = \sum_{i=0}^{n} l_i(x) f_i.$

10. Neville interpolation: $p_{i \ldots j} = \frac{(x^* - x_j) p_{i \ldots j-1} - (x^* - x_i) p_{i+1 \ldots j}}{x_i - x_j}.$

11. Divided differences: example $f[x_0, x_1, x_2] = \frac{f[x_0,x_1] - f[x_1,x_2]}{x_0 - x_2}$

    $p_n(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \cdots + f[x_0, x_1, \ldots x_n](x - x_0)(x - x_1) \cdots (x - x_{n-1})$.

12. Interpolation error for $p_n(x)$: $E(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)(x - x_1) \cdots (x - x_n)$.

13. $f' \approx \frac{\Delta f(x)}{h} = \frac{f(x+h)-f(x)}{h}$, $\quad f' \approx \frac{\nabla f(x)}{h} = \frac{f(x)-f(x-h)}{h}$, $\quad f' \approx \frac{\delta f(x)}{2h} = \frac{f(x+h)-f(x-h)}{2h}$.

14. A central-difference approximation: $f''(x_i) = \frac{f(x_i+h)-2f(x_i)+f(x_i-h)}{h^2} + O(h^2)$, $h > 0$.

15. Richardson extrapolation:
    $\Delta f(x,h) = \Delta f(x)/h \quad \text{improved} = \Delta f(x,h) + \frac{\Delta f(x,h)-\Delta f(x,2h)}{2^n-1}$
    $\delta f(x,h) = \delta f(x)/(2h) \quad \text{improved} = \delta f(x,h) + \frac{\delta f(x,h)-\delta f(x,2h)}{2^n-1}$

16. Trapezoidal rule: $\int_{x_i}^{x_{i+1}} f(x)dx = \frac{f_i + f_{i+1}}{2} h$,
    composite Trapezoidal rule: $T(h) = \frac{h}{2}(f_0 + 2f_1 + 2f_2 + \cdots + 2f_{n-1} + f_n)$, $\quad h = \frac{b-a}{n}$.

17. Simpson's 1/3 rule: $\int_{x_i}^{x_{i+2}} f(x)dx = \frac{f_i + 4f_{i+1} + f_{i+2}}{3} h$,
    composite Simpson's 1/3 rule: $S(h) = \frac{h}{3}(f_0 + 4f_1 + 2f_2 + \cdots + 4f_{n-1} + f_n)$, $\quad h = \frac{b-a}{n}$.

18. Romberg integration (trapezoidal example): improved $= T(h) + \frac{T(h)-T(2h)}{2^n-1}$,

19. Gaussian Quadrature (n-point): $\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(t_i)$ where w_i are the weights and t_i are Gaussian points.

20. Euler's method: $y_{n+1} = y_n + hf(x_n, y_n)$ to solve $\frac{dy}{dx} = f(x, y(x))$
    modified Euler's method: $y_{n+1} = y_n + \frac{h}{2}\left(f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right)$, $\quad y^*_{n+1} = y_n + hf(x_n, y_n)$.

21. A fourth-order Runge-Kutta formula to solve $\frac{dy}{dx} = f(x, y(x))$:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n) \qquad k_2 = hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1)$$
$$k_3 = hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_2) \qquad k_4 = hf(x_n + h, y_n + k_3)$$

22. Some Adams-Bashforth formulas to solve $\frac{dy}{dx} = f(x, y(x))$:

$$y_{n+1} = y_n + \tfrac{1}{24}h\left(-9f_{n-3} + 37f_{n-2} - 59f_{n-1} + 55f_n\right)$$

23. Adams-Moulton Predictor-Corrector formula to solve $\frac{dy}{dx} = f(x, y(x))$:

$$y^*_{n+1} = y_n + \tfrac{1}{24}h\left(-9f_{n-3} + 37f_{n-2} - 59f_{n-1} + 55f_n\right)$$
$$y_{n+1} = y_n + \tfrac{1}{24}h\left(f_{n-2} - 5f_{n-1} + 19f_n + 9f^*_{n+1}\right)$$