MCS 471: **SAMPLE EXAM 1** 6/28/02 NAME: ____________________

Show all work in your exam book then record answers in the space provided on this test paper. Use 4 digit *round* exam precision.

1. (a) What is the machine epsilon ε for a floating point number system with $B = 10$, precision $t = 4$, and the exponent range $L = -5$ and $U = 5$. **Soln:** 10^{-3}

   (b) Define **Relative roundoff error** and show how it is related to ε. **Soln:** $|fl(x) - x|/|x| < \varepsilon/2$, $\varepsilon/2 =$ rounding unit.

   (c) What is the smallest normalized floating point number in this system? **Soln:** $.1000 \times 10^{-5}$

2. Secant Method:

   (a) State the **Secant Method Algorithm.**

   (b) Explain why it is preferable to use the secant method instead of Newton's method. **Soln:** No derivataive is needed.

   (c) Use the *Secant method* starting with $x_1 = 0$ and $x_2 = 1$, with $|x_{k+1} - x_k| < 0.03$ as a stopping criterion, to find a root of

$$f(x) = e^x - 4x^3$$

   Algorithm: You fill this in.

k	x_k	x_{k+1}	$f(x_k)$	$f(x_{k+1})$
1	0	1	1	-1.282
2	1	.4382	-1.282	1.213
3	.4382	.7113	1.213	.5971
4	.7113	.9760	.5971	-1.065
5	.9760	.8064	-1.065	

3. Consider $f(x) = 0$ with a triple root at $x = r$.

   (a) Give conditions for $f(x)$ to have a triple root a $x = r$.
   Soln: $f(x) = (x - r)^3 g(x)$ or $f(r) = f'(r) = f''(r) = 0$ $f'''(r) \neq 0$

   (b) At what rate does the Newton-Raphson algorithm converge to the triple root at $x = r$. Justify your answer. (Use exam book or back of this sheet.)
   Soln: Writing in the form of a fixed point iteration then $g(x) = x - \frac{f(x)}{f'(x)}$ The error satisfies $e_{k+1} = g'(r)e_k$. By differentiating, we find that $g'(r) \neq 0$ so that the rate is 1.

4. Solve

$$\begin{aligned} 4.000x_1 + 10.00x_2 + 6.000x_3 &= 10.00 \\ 2.000x_1 + 8.000x_2 + 1.000x_3 &= 100.0 \\ 12.00x_1 + 16.00x_2 + 12.00x_3 &= 0.1000 \end{aligned}$$

Use Gaussian elimination, **virtual** partial pivoting and **virtual** scaling. Record the pivot vector and scale vectors at each step. Write the final augmented matrix with the multipliers (circled) stored in place of zeros. Give the final solution, the determinant and an LU decomposition.

$$\text{pivot vector } = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\text{scale vector } = \begin{bmatrix} .4000 \\ .2500 \\ .7500 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ Your choice so no interchange.}$$

Augmented Matrices:

$\boxed{.3333}$	4.667	2.000	9.967
$\boxed{.1667}$	5.333	-1.000	99.98
12.00	16.00	12.00	0.100

$\boxed{.3333}$	$\boxed{.8751}$	2.875	-77.53
$\boxed{.1667}$	5.333	-1.000	99.98
12.00	16.00	12.00	0.100

Solutions:

$x_1 =$ 8.725 $x_2 =$ 13.69

$x_3 =$ -26.97 $\det(A) =$ -184.0 (careful with sign)

$$L = \begin{bmatrix} 1 & 0 & 0 \\ .1667 & 1 & 0 \\ .3333 & .8751 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 12.00 & 16.00 & 12.00 \\ 0 & 5.333 & -1.000 \\ 0 & 0 & 2.875 \end{bmatrix}$$

5. (a) What is a sufficient condition for the Gauss-Seidel interation for $Ax = b$ to converge to the solution x?
Soln: diagonal dominance

(b) Give a Gauss-Seidel algorithm for the system below that is guaranteed to converge to the solution.

$$\begin{aligned} 4x - 8y + z &= -21 \\ -2x + y + 5z &= 15 \\ 4x - y - z &= 7 \end{aligned}$$

Soln: Interchange rows 3 and 1 then 3 and 2. You should work out the iteration strating with (0,0,0).