Math 419 - Homework 1a - C. Tier

Difference Equation Models

Read Chapter 1 in Kaplan and Glass (KG), *Understanding Nonlinear Dynamics* or Chapter 1 in Edelstein-Keshet (K-E), *Math. Models in Biology*, Answer the following questions. Do the calculations by hand (i.e. using pen and paper). You can use Maple to supplement your work.

1. Consider the growth of a strain of bacteria. Let C_t be a measure of the mass of bacteria at time t. Assume the dynamics of the bacterial growth is governed by the *linear difference equation*

$$C_{t+1} = aC_t$$

where $C_0 = 3$ and $a = 1.05$.

   (a) Generate the values of C_t for $t = 1$ to $t = 10$. Graph the result using Maple.

   (b) Describe the behavior of C_t as t gets very large.

   (c) Discuss any drawbacks of this model.

2. Linear Regression. Modify the Maple worksheet containing the in class example of linear regression, to include the following:

   (a) Fit the same data to a simple linear function, $N = C_0 + C_1 t$.

   (b) Fit the same data to the quadratic function, $N = At^2 + B$.

   (c) Graph the new functions on the same graph as the data and the exponential growth law. Please distinguish between the different curves.

3. An improved model to describe the bacterial growth is given by the *nonlinear difference equation*

$$C_{t+1} = C_t + rC_t(1 - \frac{C_t}{K}).$$

Here the parameters are $r = 0.05$, $C_0 = 3$, and $K = 14$.

   (a) Generate the values of C_t for $t = 1$ to $t = 10$. How do these values compare with the corresponding values in question 1? Graph this result.

   (b) Describe the behavior of C_t as t gets very large.

   (c) Locate the fixed points of the nonlinear difference equation and determine the local stability of each. Are these results consistent with (a) and (b)?

   (d) Discuss how this model is an improvement over the model in question 1.

4. Consider the difference equation

$$R_{t+1} - (1-f)R_t - fR_{t-1} = 0, \quad R_0 = 1, \;\; R_1 = 9/10.$$

   (a) Construct the solution for arbitrary positive f.

   (b) Give the solution if $f = .8$ and find the value of R_{12}.

   (c) Graph the solution in (b) for $t = 0 \ldots 25$. Describe what the graph indicates about the solution.

5. The Fibonacci sequence is $\{0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots\}$. Kepler observed that the terms satisfy the recursion relation (see K-E)

$$n_{k+1} = n_k + n_{k-1}.$$

   If $n_0 = 0$ and $n_1 = 1$, construct a formula for the solution of the difference equation. Kepler also observed that

$$\lim_{k\to\infty} \frac{n_{k+1}}{n_k} = \tau,$$

   where τ is the *golden mean* or *golden ratio*. Find τ. The *golden mean* appears in art, architecture, etc. (again see E-K)