Math 419 - Homework 2

Nonlinear Difference Equations

Answer the following questions, by hand when possible and also develop a separate Maple Worksheet. Be sure to include comments to clearly document your work.

1. (J.D. Murray) An experimental method to control insect populations is to introduce and maintain a number of sterile insects into the population. A simple model that has been proposed for the resulting population is

$$N_{t+1} = \frac{RN_t}{(R-1)\frac{N_t^2}{M} + N_t + S} N_t$$

where $R > 1$ and $M > 0$ are constants. The (measure of) population size is N_t and the constant sterile population size is S.

   (a) Derive the model for the population size in the absence of sterile insects (i.e. $S = 0$). Verify that when $R = 2$ the model reduces to the Beverton-Holt model discussed in class.

   (b) Sketch a graph the of the intrinsic growth multiplier (i.e. the function F if $N_{t+1} = F(N_t)N_t$) for $R = 2$ and $M = 100$. Locate the maximum multiplier. Generate a cobweb diagram if the starting population size is 25 (use 15 interates).

   (c) When $S = 0$, locate the fixed points and test the stability as done in class. Describe in words the steps in the analysis. Does it agree with the cobweb diagram above.

   (d) When $S > 0$, describe the change in the intrinsic growth multiplier curve. Explain how the introduction of the sterile insects causes this.

   (e) If we choose $R = 2$ and $M = 100$, locate the fixed points as a function of S. Test the stability.

   (f) For $S = 0.001,\ 0.01,\ 0.1,$ and 1.0. Illustrate the behavior of the population with cobweb experiments. Assume an initial size of 90. How is the long time population size effected by the size of S?

   (g) Locate a critical value S_c such that if $S > S_c$ then the insect population is eradicated. Use the parameters above and also for general parameters.

2. Human Immune System (Hoppensteadt and Peskin). A simple model of a bacterial infection has B_t measure the load of a specific bacteria in the body at time t. We assume that

$$B_{t+1} = \frac{4.0B_t}{3.0 + B_t^2} B_t.$$

The state dependent growth rate is due to immune system's response with predatory phage cells.

(a) Perform a stability analysis for the model.

(b) It is believed that there is a threshold bacterial load below which the immune system is able to eliminate the infection. Identify the threshold value in this model? Use cobweb experiments to illustrate the behavior of the model.

The following problems are for Mathematics/Computer Science Majors or for bonus credit.

3. Construct an *exact* solution to the Beverton-Holt model:

$$C_{t+1} = \frac{2}{1 + C_t/K} C_t$$

where C_0 is the mass at $t = 0$. Hint: let $C_t = 1/H_t$ and solve the new equation for H_t.

4. Generate a frequency chart or histogram for x_t in the scaled discrete logistic model with $R = 4$.