Assignment - More Differential Equations Models

1. Consider the first order chemical reaction $A \xrightarrow{k} B$

   (a) Derive a differential equation for the kinetics. Let $x(t) = [A]$.

   (b) If the half life of A is 6 hours, how long will it take for the initial concentration of A to be reduced to 10% of its initial amount.

2. You are a forensic expert who has been called in by Detective Shirley Holmes to consult on a puzzling murder. You arrive at the crime scene at 1pm and immediately measure the temperature of the body as 34.5^o C The temperature of the room at that time is 16^o C. At 2pm, you find that the temperature of body is now 33.7^o C.

   (a) Estimate the time of the murder.

   (b) Hints

   - Normal body temperature is 37^o C.
   - Assume that the temperature of the body, $T(t)$, obeys **Newton's law of cooling:** the rate of temperature change in a cooling object is proportional to the difference between the temperature of the object and the temperature T_s of the surrounding medium. This leads to the differential equation

$$\frac{dT}{dt} = k(T_s - T)$$

   (c) During the trial, the prosecution has proved and the defense agrees that the defendant arrived on the crime scene at 9:30. Hence, the prosecution claims that the defendant could have committed the crime. Is the prosecution claim feasible?

   (d) The defense claims that the your calculation is not correct since the temperature in the room was dropping linearly due to a furnace failure. Hence, the defense claims that his client arrived at the crime scene to find the victim already dead. However, your data at the crime scene includes the temperature in the room at 2pm to be 15^o C. Recalculate your result. Does the defense claim "hold water"?

3. Consider the compartment model for cold pills with one instantly dissolving dose described in class. Recall that there were two compartments, the GI tract and the bloodstream.

   (a) Solve the system of equations for $x(t)$ and $y(t)$ with the condition $y(0) = 0$ replaced by $y(0) = B$. The functions x and y describe the amount of medication, either decongestant or anithistamine, in each compartment. What do these formulas tell you about the the medication levels in the GI tract and bloodstream?

   (b) Use Maple to solve the system with $x(0) = 1$ and $y(0) = 1$. Solve for both the decongestant and antihistamine levels, i.e two sets of functions. Also, generate graphs of both x and y over a 10 hour period using the following values of k_1 and k_2.

	Decongestant	Antihistamine
k_1	1.386	0.6931
k_2	0.1386	0.0231

   Note the units are hour^{-1}. Estimate the highest level of each medication in the blood and the time when it occurs.