Math 419 - Systems of Differential Equations Assignment

Charles Tier

The following problem was adapted from the book by L. Edelstein-Keshet.

Beddington and May [1] proposed a model for the interaction between baleen whales and their main food source, krill, a small shrimp like animal. Let \( x(t) \) be the number of krill and \( y(t) \) be the number of whales. Their model is given by

\[
\begin{align*}
\frac{dx}{dt} &= rx \frac{1-x}{K} - ax y \quad \text{krill} \\
\frac{dy}{dt} &= sy \frac{1-y}{bx} \quad \text{whales}
\end{align*}
\]

The unusual feature is that the carrying capacity of the whales is a function of the number of krill (see the term \( bx \) in the denominator).

1. Using general parameters, perform a stability analysis: locate the rest points, find the Jacobian matrix, and the eigenvalues of the Jacobian matrix. Use the Maple worksheet for the predator-prey model as your starting point.

2. When is it possible for both populations to co-exist? Determine a condition on the parameters by studying the actual location of the rest points? What can you deduce about the stability under this restriction on the parameters?

3. For the specific values: \( r = 20, a = 5, K = 2, b = 3, \) and \( s = 2 \), use Maple to sketch a direction field (Use the DEplot command).

4. For the situation when there is no rest point with both populations present, what happens to both populations as time increases? Where are the rest point located?

References