Math 579 - Singular Perturbations - C. Tier

Homework 1 - Due: Wednesday, January 24, 2003

1. (Holmes, pg 6 no. 1)

   (a) What value(s) of α, if any, yield $f = O(\varepsilon^\alpha)$ as $\varepsilon \downarrow 0$

      i. $f = \sqrt{1+\varepsilon^2}$

      ii. $f = \varepsilon \sin \varepsilon$

      iii. $f = (1 - e^\varepsilon)^{-1}$

      iv. $f = \ln(1+\varepsilon)$

      v. $f = \varepsilon \ln \varepsilon$

   (b) For the functions above, what value(s) of α, if any, yield $f = o(\varepsilon^\alpha)$ as $\varepsilon \downarrow 0$?

2. Compute the asymptotic expansion, using the sequence $1,\ \varepsilon,\ \varepsilon^2, \ldots$, of $\dfrac{\tanh(1/\varepsilon)}{1+\varepsilon}$.

3. (pg 15 no. 2) Assuming $f \sim a_1\varepsilon^\alpha + a_2\varepsilon^\beta + \ldots$. Find $\alpha,\ \beta$ ($\alpha < \beta$) and nonzero a_1 and a_2 for

   (a) $f = 1/(1 - e^\varepsilon)$

   (b) $f = \sinh(\sqrt{1+\varepsilon x}),\ \ 0 < x < \infty$.

4. Consider Bessel's differential equation

$$x^2y'' + xy' + x^2y = 0$$

   with the initial conditions

$$y(0) = 1,\ \ y'(0) = 0.$$

   (a) Classify the point $x = 0$ of the ODE.

   (b) Construct the first 4 terms in the series solution of y about $x = 0$.

   (c) Evaluate the series solution at $x = 7$.

   (d) How does the solution in (c) compare with the exact solution $J_0(7)$? Maple can be used to evaluate the Bessel function

   (e) Use the asymptotic approximation on page 13 of the text to approximate the solution at $x = 7$.

   (f) Do you think that using more terms in the series solution will improve its accuracy?

5. Find a two-term asymptotic expansion, for $\varepsilon << 1$, for the solution x of $x^2 + x - \varepsilon = 0$. Compare the solutions with the exact solutions for $\varepsilon = 0.1,\ 0.01,\ 0.001$.