

Math 579 Review Problem 1

This review problem is for your own enjoyment. Please find a good ODE book and read about any the concepts that you are not familiar with.

Good Luck!

Consider

$$x^2y'' + (1 + 3x)y' + y = 0 \tag{1}$$

1. Classify *all* points of the equation including the point at ∞ .
2. Use the Frobenius method to construct a series solution to (1) in powers of x . You need to find the general term in the series.
3. What is surprising about the series solution?
4. Replace $n!$ in your series solution by Gamma integral

$$\int_0^\infty e^{-t} t^n dt.$$

Next interchange summation and integration and then sum up the series. You should now have a closed form integral.

5. Verify that the integral above is indeed a solution of (1).
6. How are the series and the integral related?