Math 584 - Applied Stochastic Models - C. Tier - Fall 2005

Homework - 2

1. Consider a discrete-state Markov chain with transition matrix

\[ A = \begin{pmatrix} 1/2 & 1/2 \\ 3/4 & 1/4 \end{pmatrix} \]

(a) Find the equilibrium state vector \( \vec{\pi} \) directly, if it exists.

(b) Find an explicit form for \( A^n \).

(c) Is it true that \( \lim_{n \to \infty} \vec{p}(0) A^n = \vec{\pi} \)?

2. Let \( A \) be an \( n \times n \) stochastic matrix. Show that the eigenvalue problems \( wA = \lambda w \) and \( Ax = \lambda x \) have the same eigenvalues but it not in general true that \( w^T = x \). For what type of matrix \( A \) are the eigenvectors transposes of each other?

3. A Markov chain with state-space \( \{0,1,2\} \) has transition matrix

\[ A = \begin{pmatrix} .4 & .2 & .4 \\ .6 & 0 & .4 \\ .2 & .5 & .3 \end{pmatrix} \]

(a) Sketch the transition diagram.

(b) Based on the transition diagram can you conclude that \( A \) is irreducible and aperiodic.

You might use the Perron-Frobenius theory to make the same conclusion.

(c) Use Matlab or Maple to compute \( A^{15} \).

(d) Find the equilibrium state vector \( \vec{\pi} \) directly, if it exists. Is the result consistent with (c)?

4. Let \( X(n) \) be the location of a random walk on the state-space \( S = \{0, 1, \ldots, N\} \). The state 0 is absorbing and the state \( N \) is reflecting. At the states \( 1, \ldots, N - 1 \), the random walk can jump left with probability \( l \) or jump right with probability \( r \), where \( r + l = 1 \).

(a) Draw the transition diagram and state the transition matrix \( A \).

(b) Find the equilibrium solution, if possible.

(c) Let \( u_k = \Pr[X(n^*) = 0 | X(0) = k] \). Derive a problem for \( u_k \) involving the backward equation and solve it. Here \( n^* \) is the first time (first passage time) that \( X \) reaches 0.

(d) Let \( T_k = E[n^* | X(0) = k] \). Derive a problem for \( T_k \) involving the backward equation and solve it. Here \( T_k \) is referred to as the mean first passage time.