Homework 3

MATH 215 (due September 16) September 9, 2022

Problem 1. Apply each of the following claims to two specific examples of your choice or find a counterexample. In your solution, you should provide the examples and what you have concluded from the statements:

a. Suppose that n is a integer, such that 6 divides $n(n+1)(n+2)$ then 24 divides $n(n+1)(n+2)(n+3)$.

b. Suppose that x, y, z are three integers such that $x^2 + y^2 = z^2$, then either 3 divides x or 3 divides y.

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Problem 2. Prove the following equivalences (using a double implication):

a. An integer is divisible by 5 if and only if its last digit is divisible by 5.

   [Hint: To formally refer to the unit number of an integer n, decompose $n = 10k + d$ where k is some integer and $0 \leq d \leq 9$. Then d is the unit digit of n.]

b. An integer is divisible by 4 if and only if its last two digits form a number divisible by 4.

   [Hint: Decompose $n = 100l + d$ where k, l is some integers and $0 \leq d \leq 99$. Then the number d is the last two digits.]

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Problem 3. In each of the following items, determine if the conclusion logically follows from the premises and prove your answer.

a. **Pre. 1:** $A \wedge (B \Rightarrow \neg C)$

   Pre. 2: $B \vee C$

   Conclusion: $\neg C$

b. **Pre. 1:** $A \Rightarrow (B \vee (\neg C))$

   Pre 2. $A \wedge C$

   Pre 3. $D \Rightarrow \neg B$

   Conclusion: $\neg D$

Problem 4.

a. Prove that for all integers n and m, if n is multiple of 6 or m is multiple of 9 then n^2m is a multiple of 9.

b. Prove that if a and b are odd integers, then $a^2 - b^2$ is a multiple of 8.

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Problem 5. Prove that for all integers a, b, c if $a^2 + b^2 = c^2$, then abc is even.