Homework 6

MATH 215 | (due October 14) | October 7, 2022

Problem 1. Describe the set $P(\{\emptyset, \{\emptyset\}\})$ using the list principle. No proof required.

Problem 2. Prove that for every two sets A, B the following are equivalent:

- $A \subseteq B$.
- $P(A \cup B) = P(B)$.
- $P(A) \subseteq P(B)$.

[Remember: You are allowed to use the propositions and statements which appear in the class notes.]

Problem 3. Prove the following statements by induction:

1. For every $n \geq 2$, $n! < n^n$.

2. For every natural number n, the number $17n^3 + 103n$ is multiple of 6.

Problem 4. Prove that for every natural number n,

$$1 + 4 + 4^2 + \cdots + 4^n = \frac{4^{n+1} - 1}{3}.$$

Problem 5. Prove that for all odd natural numbers n, the number $2^n + 1$ is multiple of 3.

[Hint: Express n as $2k + 1$ and preform induction on k.]

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Problems 6,7 are optional.

Problem 6. Spot the problem in the following obviously wrong argument:

> «All horses are the same color. Specifically, every finite set of horses is monochromatic.»
>
> *Proof.* We argue by induction. The statement is clearly true for sets of size 1. Assume by induction that all sets of n horses are monochromatic, and consider a set of size $n + 1$. The first n horses are all the same color. The last n horses are all the same color. Because of the overlap, this means that all $n + 1$ horses are the same color. So by induction, all finite sets of horses are all the same color, and so all horses are the same color. $\square$

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Problem 7. Suppose $x \geq -1$. Prove that for all natural number n,

$$(1 + x)^n \geq 1 + nx.$$