Homework 8

MATH 215 (due October 28) October 21, 2022

Problem 1. Compute the following gcd's using the Euclidean algorithm:

1. $gcd(46, 112)$.

2. $gcd(426, 252)$.

3. $gcd(142, 235)$.

Homework 8

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Problem 2. Prove the following claims:

1. For any integers n_1, n_2 and $m > 0$ $n_1 \equiv n_2 (\mathrm{mod} m)$ if and only if $n_1 - n_2$ is divisible by m.

2. For every integers n and $m > 0$, $n \equiv n \mathrm{mod} m (\mathrm{mod} m)$

Homework 8

Problem 3. Prove that for any non-zero integers n_1, n_2:

1. $1 \leq gcd(n_1, n_2) \leq n_1, n_2$.

2. $gcd(n_1, n_2) = n_1$ if and only if n_1 divides n_2.

Homework 8

Problem 4. Prove that for any integers n_1, n_2, m, where $m > 0$,

$$n_1 \mathrm{mod} m = 1 \Rightarrow n_1 \cdot n_2 \equiv n_2 (\mathrm{mod} m).$$

Problem 5. Prove that for every natural number n, n^2 is divisible by 25 if and only if n is divisible by 5.

[Hint: Use the exercise we saw in class that n is divisible by 5 iff n^2 is divisible by 5.]