# Mathematical Reasoning-Finals Example

MATH 300 (Instructor: Tom Benhamou)

## Instruction

The midterm consists of 5 problems, each worth 21 points (The maximal grade is 100). The duration of the exam is 3 hours. No external material/equipment is authorized. You can only rely on statements we have seen in class and proof techniques we have presented in class. The answers to the problems should be answered in the designated areas.

## Problems

**Problem 1.** Compute the following set and determine whether it is countable, prove your answer: $\{x \in \mathbb{R} \mid |[x - \frac{1}{2}, x + \frac{1}{2}] \cap \mathbb{Z}| = 3\}$.

**Solution:**

**Problem 2.** Prove that for every natural number $n \geq 1$ we have:

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

**Solution:**

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**Problem 3.** Answer the following items, no proof required:

a. Determine whether the formula $(A \vee B) \Rightarrow ((\neg A) \vee (B \rightarrow A))$

is a tautology / is **not** a tautology

b. Determine whether $\{y \in \mathbb{N} \mid y \cdot y = y\} \in \{X \in P(\mathbb{N}) \mid |X| = 2\}$

is true / is false

[Recall: $|X|$ stands for the number of elements in the set $X$.]

c. Consider the statements

$$\alpha_1 = (A \wedge B) \Rightarrow C, \ \alpha_2 = C \wedge A \text{ and } \alpha = A \vee \neg B$$

Determine whether the conclusion $\alpha$:

logically follows from $\alpha_1, \alpha_2$ / **does not** logically follow from $\alpha_1, \alpha_2$.

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**Problem 4.** Prove that if $\sqrt{7}$ and $\sqrt{28}$ are irrational.

**Solution:**

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**Problem 5.** Let $E = \{\langle A, B\rangle \in P(\mathbb{Z})^2 \mid \exists k \in \mathbb{Z},\ A + k = B\}$ where $A + k = \{a + z \mid a \in A\}$.

1. Prove that $E$ is an equivalence relation on $P(\mathbb{Z})$.

2. Compute $|[\mathbb{Z}]_E|, |[\mathbb{N}]_E|$, no proof required.

**Solution:**