

## Homework 9

MATH 300

(due April 12)

April 5, 2024

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**Problem 1.** Prove that if  $A \sim B$  and  $B \sim C$  then  $A \sim C$ .

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**Problem 2.** Prove the following items:

1.  $\mathbb{N} \setminus \{2023, 2024\} \sim \mathbb{N}_{\text{even}}$ .

2.  $P(\mathbb{N}) \setminus \{\emptyset\} \sim P(\mathbb{N})$ .

3.  $(0, 1) \sim (0, \infty)$ .

4.  $\mathbb{Z} \times [0, 1) \sim \mathbb{R}$ .

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**Problem 3.** Prove that for every  $\alpha < \beta$  real numbers  $(\alpha, \beta) \approx (0, 1)$ . [Hint: First stretch/shrink  $(0, 1)$  to have length  $\beta - \alpha$ , then shift it by  $+c$  as we did in class.]

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**Problem 4.** Show that  $\mathbb{N}\{0, 1\} \times \mathbb{N}\{0, 1\} \approx \mathbb{N}\{0, 1\}$ .

[Hint: Use the interleaving function  $F : (\mathbb{N}\{0, 1\})^2 \rightarrow \mathbb{N}\{0, 1\}$  defined by

$$F(\langle f, g \rangle)(n) = \begin{cases} f(\frac{n}{2}) & n \in \mathbb{N}_{\text{even}} \\ g(\frac{n-1}{2}) & n \in \mathbb{N}_{\text{odd}} \end{cases}$$

as the witnessing bijection.]