Math 300
Midterm Examples

February 16, 2024

Instruction

The midterm consists of 3 problems, each worth 34 points (The maximal grade is 100). For this you will have 45 minutes during class. The identities file will be appended to the exam and no other material is allowed. The answers to the problems should be answered in the designated areas.

Examples for problems

Problem 1. Determine whether "$(A \vee (A \Rightarrow B)) \Rightarrow (A \wedge B)$" is a tautology. Justify your answer using truth tables:

Problem 2. Determine whether the conclusion logically follows from the premises:

Premise 1: $A \wedge (B \Rightarrow C)$

Premise 2: $A \vee B$

Conclusion: $\neg C$

Problem 3. Let α be the statement:

$$\forall x \in \mathbb{Z}.\Big(\forall y \in \mathbb{Z}.\big((x < y) \Rightarrow (\exists z \in \mathbb{Z}.x < z + 1 < y)\big)\Big)$$

a. present $\neg\alpha$ without the "$\neg$" symbol.

b. Prove or disprove α.

Problem 4. Prove that if $2a - b$ is multiple of n and $a - b$ is multiple of n, then a is multiple of n.

Problem 5. Prove that if $n^2 + 2n + 11$ is odd, then n is even.

Problem 6. Express the following sets using the list principle. No proof required.

1. $\{x \in \mathbb{N} \mid x \cdot 5 \leq 5\}$
2. $\{x \in \mathbb{Z} \mid |x| < 10 \wedge \exists y.5y = x\}$.
3. $\left\{\{x^2, |x|\} \mid x \in \{-1, 0, 1, 2\}\right\}$

Problem 7. Prove the following statements:

1. $\frac{1}{2} \in \{q + 1 \mid q \in \mathbb{Q}\}$.
2. $3 \notin \{x \in \mathbb{Z} \mid x^2 + 2x + 1 = 0\}$
3. $5 \in \left\{n \in \mathbb{N} \mid n^2 \in \{2m + 1 | m \in \mathbb{N}\}\right\}$