Mathematical Reasoning-Midterm 1

MATH 300 (Instructor: Tom Benhamou) February 26, 2024

Instruction

The midterm consists of 3 problems, each worth 34 points (The maximal grade is 100). For this you will have one hour. The identities file will be appended to the exam and no other material is allowed. The answers to the problems should be answered in the designated areas.

Problems

Problem 1. For each of the following statements determine if it is true are false. No explanation required, circle the correct answer:

a. $\{2,1\} \in \left\{1,2,\{1,1,2\},\{\{1\}\}\right\}$. <u>True</u> \ False

b. $0 \in \left\{x^2 - x - 2 \mid x \in (0,2)\right\}$. True \ <u>False</u>

c. $6 \in \left\{n \in \mathbb{N}_+ \mid \{m \in \mathbb{N} \mid 1 < m < n \wedge n \text{ is divisible by } m\} = \emptyset\right\}$.
True \ <u>False</u>

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Problem 2. Prove that for all positive integers a, b, if both $4a+b$ and $4b-3a$ are divisible by 5 then a is divisible by 5.

Solution: Suppose that $4a + b$ and $4b - 3a$ are divisible by 5. WTP 5 divides a. We have that $(4a + b) + (4b - 3a) = a + 5b$ and therefore $a = (4a + b) + (4b - 3a) - 5b$. Since $4a + b, 4b - 3a, -5b$ are divisible by 5, they sum is divisible by 5 and therefore a is divisible by 5.

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Problem 3. Let α be the statement:

$$\forall n \in \mathbb{N}_+ \Big(\forall y \in \mathbb{Z}(\exists q \in \mathbb{Q}((y < q) \wedge (q < y + \frac{1}{n^2})))\Big)$$

a. Present $\neg\alpha$ without the "$\neg$" symbol. No explanation required.

Solution:

$$\neg\alpha \equiv \underline{\exists n \in \mathbb{N}_+ \Big(\exists y \in \mathbb{Z}(\forall q \in \mathbb{Q}((y \geq q) \vee (q \geq y + \tfrac{1}{n^2})))\Big)}$$

b. Prove or disprove α.

solution: Let us prove α, let $n \in \mathbb{N}_+$ and $y \in \mathbb{Z}$. WTP $\exists q \in \mathbb{Q}$ such that $y < q < y + \frac{1}{n^2}$. Define $q = y + \frac{1}{n^2+1}$, then $q \in \mathbb{Q}$ (since $y \in \mathbb{Q}$ and $\frac{q}{n^2+1} \in \mathbb{Q}$ and the sum of rationals is rational). Since $\frac{1}{n^2+1} > 0$ we have that $y < y + \frac{1}{n^2+1} = q$. Also, since $n^2 + 1 > n^2$, then $\frac{1}{n^2+1} < \frac{1}{n^2+1}$ and therefore $q = y + \frac{1}{n^2+1} < y + \frac{1}{n^2}$.