Math 300 Intro Math Reasoning
Worksheet 04: Mathematical logic

(1) Compute the negation and prove or disprove the following statement.

$$\forall x(\forall y((x < y) \Rightarrow (\exists z(x < z \wedge z < y)))).$$

(2) Prove that $1 + \sqrt{2}$ is irrational.

(3)

Prove that $\sqrt{3}$ is irrational.

(4) $A = \{1, 2, 3\},\ B = \{1, 1, 2, 3\},\ C = \{n \in \mathbb{N} \mid \exists y \in \mathbb{R}(|y| + |3 - n| \leq 3)\}$,
$D = \{\{1\}, \{1, 2\}, \{1, 2, 3\}\},\ E = \{1, \{1, 2, 3\}, 3\}\ F = \{2^n - m \mid n \in \mathbb{N}, m \in \{0, 1\}\}$

(1) How many elements are in each of the sets?
(2) Determine if
   (a) $A = B$.
   (b) $A \subseteq E$.
   (c) $A \in E$.
   (d) $A = C$.
   (e) $A \subseteq C$
   (f) $E \subseteq D$.
   (g) $A \subseteq F$
   (h) $C \subseteq F$?