Math 300 Intro Math Reasoning
Worksheet 08: Equinnumerability

(1)

(1) Let $Y = \{n + 2 \mid n \in \mathbb{N}\} \subseteq \mathbb{N}$. Find a bijective map $f : \mathbb{N} \longrightarrow Y$.
(2) Show that $(-1, 2) \sim (6, 7)$
(3) Find an injection from $\mathbb{N} \times \mathbb{N}$ into $P(\mathbb{N})$.

Solution.

(1) $f(n) = n + 2$.
(2) $f(x) = \frac{x+1}{3} + 6$.
(3) $f : \mathbb{N} \times \mathbb{N} \to P(\mathbb{N})$ $f(\langle n, m\rangle) = \{n, n + m\}$.

(2) Suppose that $A \sim B$ and $C \sim D$. Prove that $A \times C \sim B \times D$

Solution. Let $f : A \to B$ and $g : C \to D$ be bijections. Define $h : A \times C \to B \times D$ $h(\langle a, c\rangle) = \langle f(a), g(c)\rangle$. Prove that h is one-to-one. Let us prove for example that h is onto. Let $\langle b, d\rangle \in B \times D$. Since f, g are onto, there are $a \in A$ and $c \in C$ such that $f(a) = b$ and $g(c) = d$. Then $\langle a, c\rangle \in A \times C$ and $h(\langle a, c\rangle) = \langle f(a), g(c)\rangle = \langle b, d\rangle$.