Homework 9

MATH 361 (due December 8) November 25, 2023

Problem 1. In this exercise we consider the statement α: "Suppose that $\mathcal{A}$ is a set of open pairwise disjoint intervals in $\mathbb{R}$, then $|\mathcal{A}| \leq \aleph_0$."

(1) Prove α: for every $\emptyset \neq I \in \mathcal{A}$ there is a rational $q_I \in I \cap \mathbb{Q}$ (why?), the define $f(I) = q_I$ and prove that f is injective.

(2) Did you use the axiom of choice in the previous proof? if so, where did you use it?

Problem 2. Show that if A is countable then there is a choice function for $P(A) \setminus \{\emptyset\}$

Problem 3. Let SRP be the principle that for every set A and for every equivalence relation E on A, there exists a system of representatives.

Prove that the Axiom of Choice is equivalent to SRP.

1 Additional problems

Problem 4. The principle of Dependent choice (DC) is the following: For every total relation R on A there is a sequence $(x_n)_{n\in\mathbb{N}} \subseteq A$ such that for every n. $x_n R x_{n+1}$. Prove that DC follows from AC.

Remark: It is known that DC does not imply AC. For more information about DC see Link