Homework 1

MATH 461 (due February 2) January 26, 2024

Problem 1 (optional)**.** Prove that if A, B, C are sets then

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Problem 2. Let $\mathcal{B}$ be a nonempty set of sets and let A be any set. Show that

(a) $A \cap \bigcup \mathcal{B} = \bigcup\{A \cap B \mid B \in \mathcal{B}\}$.

(b) $A \setminus \bigcap \mathcal{B} = \bigcup\{A \setminus B \mid B \in \mathcal{B}\}$.

Problem 3 (optional)**.** For a function $f : A \to B$ and $C \subseteq A$ define the *pointwise image of* C *by* f as

$$f''C = \{f(c) \mid c \in C\}$$

(a) Prove that if $f : A \to B$ is a function and $C \subseteq A$, then

$$(f''A) \setminus (f''C) \subseteq f''[A \setminus C].$$

(b) Give an example of a function $f : A \to B$ and a subset $C \subseteq A$ such that

$$(f''A) \setminus (f''C) \neq f''[A \setminus C].$$

(c) Prove that if $f : A \to B$ is an injection and $C \subseteq A$, then

$$(f''A) \setminus (f''C) = f''[A \setminus C].$$

Problem 4. Recall that the indicator function $\chi_A : P(A) \to {}^A\{0, 1\}$ is defined by $\big(\chi_A(B)\big)(a) = \begin{cases} 1 & a \in B \\ 0 & a \notin B \end{cases}$. Prove that χ_A is injective.

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Problem 5. Prove that the interleaving function $F : ({}^{\mathbb{N}}\{0,1\})^2 \to {}^{\mathbb{N}}\{0,1\}$ defined by

$$F(\langle f, g\rangle)(n) = \begin{cases} f(\frac{n}{2}) & n \in \mathbb{N}_{even} \\ g(\frac{n-1}{2}) & n \in \mathbb{N}_{odd} \end{cases}$$

is one-to-one and onto. Prove that it is invertable and find F^{-1}.

Problem 6. Prove the following statements:

(a) $\left\{f \in {}^{\mathbb{R}}\mathbb{R} \mid \exists i \in \{0,1\}, \forall x \in \mathbb{R} \setminus \mathbb{Q}, f(x) = i\right\} \approx \{0,1\} \times {}^{\mathbb{Q}}\mathbb{R}$.

(b) If $A \approx B$ then $P(A) \approx P(B)$