Problem 1. Prove that if α , β are any WFF's, then

$$(\forall x(\alpha \to \beta) \to (\forall x\alpha \to \forall x\beta))$$

is valid.

Problem 2. Prove that if *P* is a binary predicate symbol, then

$$(x=y \to (P(x,z) \to P(y,z)))$$

is valid.

Problem 3. Prove that if $\Gamma \vdash \alpha_1$ and $\Gamma \vdash \alpha_1 \rightarrow \alpha_2$ then $\Gamma \vdash \alpha_2$

Problem 4. Show that $\vdash \exists v_1 P(v_1) \rightarrow \exists v_2 P(v_2)$

[Small Hint: Start your deduction with $\forall v_2 \neg P(v_2) \rightarrow \neg P(v_1)$ (justify

it!). At some point you should use the generalization theorem.]