Introduction to advanced mathematics
2nd Midterm Examples

October 29, 2022

Instruction

The midterm consists of 3 problems, each worth 34 points (The maximal grade is 100). For this you will have 45 minutes during class. The identities file will be appended to the exam and no other material is allowed. The answers to the problems should be answered in the designated areas.

Examples for problems

Problem 1. Prove that $(A \setminus B) \setminus C = (A \setminus C) \setminus (B \setminus C)$.

Problem 2. Prove by induction that for every $n \in \mathbb{N}_+$, $1+3+\ldots+(2n-1) = n^2$.

Problem 3. Prove that for every integer $n > 0$, $n, n+1$ are coprime.

Problem 4. Prove that for all $n \in \mathbb{N}$, $9^n - 5^n$ is divisible by 4.

Problem 5. Express the following sets using the list principle. No proof required.

1. $(-5, 5) \cap \mathbb{Z}$.
2. $\{\emptyset, 1\} \times \{n \in \mathbb{N} \mid |P(\{1, \ldots, n\})| < 4\}$.
3. $\{\langle x, y\rangle \in \mathbb{R}^2 \mid x^2 + y^2 = 1\} \cap \{\langle x, x\rangle \mid x \in \mathbb{R}\}$.

Problem 6. Compute the following

1. Compute A_3, where A_n is defined recursively by $A_0 = \emptyset$ and $A_{n+1} = A_n \cup \{A_n\}$.
2. a_4 where a_n is defined recursively by $a_0 = 0$ and $a_{n+1} = 2^{a_n}$
3. a_{100} where a_n is defined recursively by $a_0 = 2$, $a_1 = 3$ and $a_{n+1} = gcd(a_n, a_{n-1}) + 1$.