Answer Key

Math 180
Worksheet 12
Section 4.6-4.7

1.) Consider $f(x) = 1 - |x|$ on the interval $[-1,1]$. Show that f does not reach the conclusion of Rolle's Theorem. Which of the conditions does it fail?

$f'(x) = \begin{cases} x & \text{if } x<0 \\ -x & \text{if } x>0 \end{cases}$ $f'(0)$ does not exist nor does $f'(1)$ or $f'(-1)$

Thus $f'(x) \neq 0$ for all x even though $f(-1) = f(1) = 0$

It fails the condition that it be differentiable on $(-1,1)$

2.) Show that the function $f(x) = x + \frac{1}{x}$ on the interval $[2,3]$ satisfies the conditions of the mean value theorem. Then find all numbers c in the interval that reach the conclusion of the theorem.

$f(x)$ is defined & continuous on $[2,3]$

$f'(x) = 1 - \frac{1}{x^2}$ so f is differentiable on $(2,3)$

So f satisfies the conditions of the M.V.T.

thus there exists c in $(2,3)$ such that

$f'(c) = \frac{f(3)-f(2)}{3-2} = \frac{(3+\frac{1}{3})-(2+\frac{1}{2})}{3-2} \cancel{= \frac{(\frac{4}{3}-\frac{3}{2})}{1} = \frac{8}{6} - \frac{9}{6}}$

$\cancel{= \frac{\frac{10}{3}-\frac{3}{2}}{1} = \frac{20}{6} - \frac{9}{6} = \frac{11}{6}}$ $= \frac{\frac{10}{3}-\frac{5}{2}}{1} = \frac{20}{6} - \frac{15}{6} = \frac{5}{6}$

$\cancel{\frac{11}{6} = 1 - \frac{1}{c^2} \Rightarrow \frac{1}{c^2} = 1 - \frac{11}{6} = \frac{-5}{6}}$ $1 - \frac{1}{c^2} = \frac{5}{6} \Rightarrow \frac{1}{c^2} = \frac{1}{6} \Rightarrow 6 = c^2$

$\Rightarrow c = \pm\sqrt{6} \Rightarrow \boxed{c = \sqrt{6}}$

3.) For what ~~intermediate~~ indeterminate forms can L'Hôpital's rule be applied directly (without algebraic manipulation)?

$\frac{\infty}{\infty}$ or $\frac{0}{0}$

4.) If possible, use L'Hôpital's rule to find the following limits.

a. $\lim_{x \to 1} \frac{x-1}{x^2-1}$ $= \lim_{x \to 1} \frac{1}{2x} = \frac{1}{2}$

$\frac{0}{0}$

b. $\lim_{x \to 0} \frac{e^{3x}-1}{x}$ $= \lim_{x \to 0} \frac{3e^{3x}}{1} = 3e^{3 \cdot 0} = 3$

$\frac{0}{0}$

5.) If possible, use L'Hôpital's rule to find the following limits.

a. b. $\lim_{x\to 0} x\cot x = \lim_{x\to 0} \frac{x\cos(x)}{\sin(x)} = \lim_{x\to 0} \frac{x}{\tan(x)} = \lim_{x\to 0} \frac{1}{\sec^2(x)} = \lim_{x\to 0} \cos^2(x) = 1^2 = 1$

$\frac{0}{0}$

b. $\lim_{x\to\infty} \left(x - \sqrt{x^2+5x}\right) = \lim_{x\to\infty} \left(x - \sqrt{x^2+5x}\right)\left(\frac{x+\sqrt{x^2+5x}}{x+\sqrt{x^2+5x}}\right) = \lim_{x\to\infty} \frac{x^2 - x^2 - 5x}{x+\sqrt{x^2+5x}}$ $\cancel{\lim \frac{5x}{x+\sqrt{x^2+5x}}}$

$= \lim_{x\to\infty} \frac{-5x}{x+\sqrt{x^2+5x}}$ $\cancel{= \lim_{x\to\infty} \frac{5}{1+\frac{2x+5}{\sqrt{x^2+5x}}\cdot\frac{1}{2}} = \frac{10\sqrt{x^2+5x}}{\sqrt{x^2+5x}+2x+5}}$

$\frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{-5}{1+\sqrt{1+\frac{5}{x}}} = \frac{-5}{1+\sqrt{1}} = \frac{-5}{2}$

c. $\lim_{x\to 1} x^{1/(1-x)} = L$

$\ln(L) = \ln\left(\lim_{x\to 1} x^{\frac{1}{1-x}}\right) = \lim_{x\to 1} \ln\left(x^{\frac{1}{1-x}}\right) = \lim_{x\to 1} \frac{1}{1-x}\ln(x) = \lim_{x\to 1} \frac{\ln(x)}{1-x}$

$\frac{0}{0}$

$= \lim_{x\to 1} \frac{\left(\frac{1}{x}\right)}{-1} = \lim_{x\to 1} \frac{-1}{x} = \frac{-1}{1} = -1$

$\ln(L) = -1$

$L = e^{\ln(L)} = e^{-1}$

so $\lim_{x\to 1} x^{\frac{1}{1-x}} = \frac{1}{e}$