

Answer Key

Math 180

Worksheet 13

Section 4.8, 5.1-5.2

1.) Find all the antiderivatives of the following functions. Check your work by taking derivatives.

a. $f(x) = 5x^4$

$$\int 5x^4$$

b. $g(x) = \sin(2x)$

$$-\frac{1}{2} \cos(2x)$$

c. $P(t) = -6t^{-7}$

$$t^{-6}$$

2.) Determine the following indefinite integral. Check your work by differentiation.

a. $\int (2x^5 - 3x^8) dx = \frac{2}{6}x^6 - \frac{3}{9}x^9 + C = \frac{1}{3}x^6 - \frac{1}{3}x^9 + C$

b. $\int (\sin(2u) + \cos(3u)) du = -\frac{1}{2} \cos(2u) + \frac{1}{3} \sin(3u) + C$

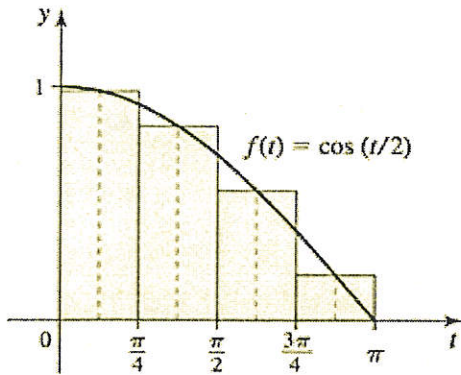
c. $\int \left(\frac{3}{4+y^2}\right) dy = 3 \int \frac{dy}{y^2+2} = \frac{3}{2} \int \frac{dy}{1+\left(\frac{y}{\sqrt{2}}\right)^2} = \frac{3}{2} \int \frac{1}{1+u^2} du$
 $= \frac{3}{2} \tan^{-1}\left(\frac{y}{\sqrt{2}}\right) + C$

(a) $\frac{d}{dx} \left(\frac{1}{3}x^6 - \frac{1}{3}x^9 + C \right) = 2x^5 - 3x^8$

(b) $\frac{d}{du} \left(-\frac{1}{2} \cos(2u) + \frac{1}{3} \sin(3u) + C \right) = \sin(2u) + \cos(3u)$

(c) $\frac{d}{dx} \left(\frac{3}{2} \tan^{-1}\left(\frac{y}{\sqrt{2}}\right) + C \right) = \frac{3}{2} \cdot \frac{1}{2} \frac{1}{1+\left(\frac{y}{\sqrt{2}}\right)^2} = \frac{3}{4+y^2}$

- 3.) Approximate the area of the region bounded by the graph of $f(t) = \cos(t/2)$ and the t -axis on $[0, \pi]$ with $n = 4$ subintervals. Use the midpoint of each subinterval to determine the height of the rectangle (see figure). (Use your calculators for the y -values)



$$\text{Area} = A$$

$$A \approx \left(\frac{\pi-0}{4}\right) \left(\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{5\pi}{8}\right) + \cos\left(\frac{7\pi}{8}\right)\right)$$

$$= \frac{\pi}{4} \left(\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{5\pi}{8}\right) + \cos\left(\frac{7\pi}{8}\right)\right)$$

$$= \frac{\pi}{4} (.98 + .83 + .56 + .20)$$

$$= \frac{\pi \cdot 2.57}{4} \approx 2.02$$

- 4.) Express the sums in Sigma Notation.

a. $1 + 2 + 3 + 4 + 5 + 6 + 7$

$$\sum_{i=1}^7 i$$

b. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

$$\sum_{i=1}^5 \frac{1}{i}$$

5.) Evaluate the following first by hand, and then by using the appropriate formulas.

a. $\sum_{k=1}^8 k$ b. $\sum_{k=1}^5 (2k + 1)$

(a) $1+2+3+4+5+6+7+8 = 36$
 $\frac{8+18}{2} \cdot 8 = 94 = 36$

(b) $3+5+7+9+11 = 35$
 $2 \cdot \frac{5+15}{2} + 5 = 30 + 5 = 35$

6.) Use only the fact that $\int_0^4 3x(x-2)dx = 16$ and the definite integral properties to calculate the following:

a. $\int_4^0 6x(x-2)dx =$

~~$\int_0^4 (6x^2 - 12x)dx$~~
 $= -2 \int_0^4 3x(x-2)dx$
 $= -2(16) = -32$

b. $\int_0^4 3x(2-x)dx =$

$= - \int_0^4 3x(x-2)dx$
 $= -16$