

# Answer Key

Math 180  
Worksheet 2  
Section 2.3

1.) How is  $\lim_{x \rightarrow a} f(x)$  calculated if  $f$  is a polynomial function?

$$\lim_{x \rightarrow a} f(x) = f(a) \text{ if } f \text{ is a poly. func.}$$

2.) Evaluate the following limits:

a.  $\lim_{x \rightarrow -4} 4$

$$= \boxed{4}$$

b.  $\lim_{x \rightarrow 0.5} (5x^3 + 2x - 1)$

$$\begin{aligned} &= 5\left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right) - 1 \\ &= 5\frac{1}{8} + 1 - 1 = \boxed{\frac{5}{8}} \end{aligned}$$

c.  $\lim_{x \rightarrow 3} \frac{x+2}{x-5}$

$$= \frac{3+2}{3-5} = \boxed{\frac{5}{-2}}$$

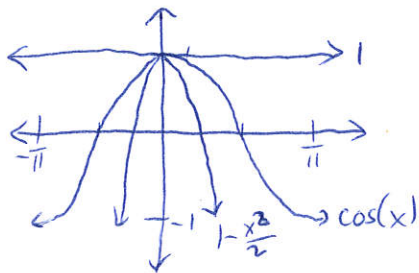
3.) Evaluate the following limits:

a.  $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x \rightarrow 3} \frac{x+3}{1} = \frac{3+3}{1} = \boxed{6}$

$$\begin{aligned}
 \text{b. } \lim_{h \rightarrow 0} \frac{\sqrt{16+h}-4}{h} &= \lim_{h \rightarrow 0} \frac{\sqrt{16+h}-4}{h} \cdot \frac{(\sqrt{16+h}+4)}{(\sqrt{16+h}+4)} = \lim_{h \rightarrow 0} \frac{(\sqrt{16+h})^2 + 4\sqrt{16+h} - 4\sqrt{16+h} - 16}{h(\sqrt{16+h}+4)} \\
 &= \lim_{h \rightarrow 0} \frac{16+h-16}{h(\sqrt{16+h}+4)} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{16+h}+4)} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{16+h}+4} \\
 &= \frac{1}{\sqrt{16+0}+4} = \frac{1}{4+4} = \boxed{\frac{1}{8}}
 \end{aligned}$$

4.) It can be shown that  $1 - \frac{x^2}{2} \leq \cos x \leq 1$ , for  $x$  near 0.

a. Illustrate these inequalities with a graph.



b. Use these inequalities to evaluate  $\lim_{x \rightarrow 0} \cos x$ .

$$\lim_{x \rightarrow 0} \left(1 - \frac{x^2}{2}\right) = 1 - \frac{0^2}{2} = 1 - 0 = 1$$

$$\lim_{x \rightarrow 0} 1 = 1$$

~~So by squeeze thm,~~

So by squeeze thm,  $\lim_{x \rightarrow 0} \cos(x) = 1$