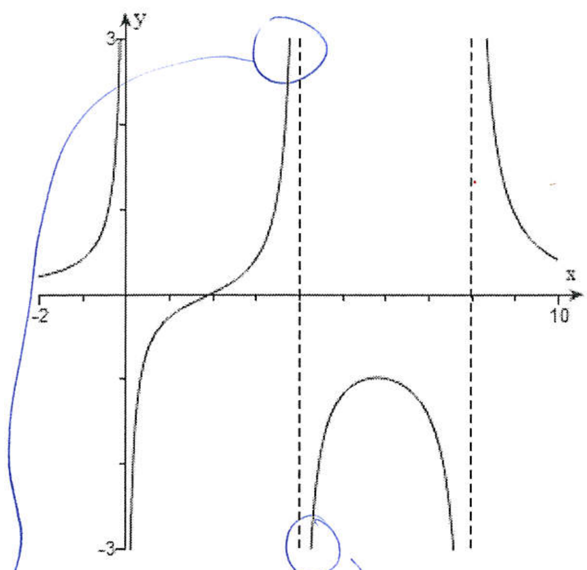
Answer Key

Math 180
Worksheet 3
Section 2.4-2.6, 3.1

1.) The graph below of the function h has vertical asymptotes $x = 0, x = 4$, and $x = 8$. Analyze the following limits.



a. $\lim_{x \to 4^-} h(x)$

∞

b. $\lim_{x \to 4^+} h(x)$

$-\infty$

c. $\lim_{x \to 4} h(x)$

D.N.E.
(since $\lim_{x \to 4^-} h(x) \neq \lim_{x \to 4^+} h(x)$)

2.) Determine whether the following statements are true or false. If true, explain why. If false, give a counterexample.

a. The line $x = 1$ is a vertical asymptote of the function $f(x) = \frac{x^2-7x+6}{x^2-1}$.

$f(x) = \frac{(x-6)(x-1)}{(x+1)(x-1)} = \frac{x-6}{x+1}$

So $\boxed{\text{False}}$ because $x=1$ is a hole as it cancels out of the denominator

b. If g has a vertical asymptote at $x = 1$ and $\lim_{x \to 1} g(x) = \infty$, then $\lim_{x \to 1^+} g(x) = \infty$. True, $\lim_{x \to 1} g(x) = \infty \Rightarrow$ implies $\lim_{x \to 1^+} g(x) = \infty$

3.) Determine the following limits:

a. $\lim_{x \to \infty} \left(3 + \frac{10}{x^2}\right)$

$= 3 + 0$

$= 3$

Since $\lim_{x \to \infty} \frac{10}{x^2}$

$= 10\left(\lim_{x \to \infty}\left(\frac{1}{x}\right)\right)^2$

$= 10 \cdot 0^2 = 0$

b. $\lim_{x \to -\infty} (-3x^{16} + 2)$

$= -\infty$

(Since $\lim_{x \to -\infty}(-3x^{16}+2)$

$= -3\left(\lim_{x \to -\infty} x\right)^{16} + \lim_{x \to -\infty} 2$

$= -3(\infty) + 2 = -\infty + 2 = -\infty$)

4.) Consider the rational function $f(x) = \frac{x^2-4x-21}{x^2+9x+14}$. Find vertical, horizontal, and oblique asymptotes (if there are any). Then sketch the graph (include intercepts and other points to help you with the graph). Check your answer with your calculator.

$f(x) = \frac{(x-7)(x+3)}{(x+2)(x+7)}$

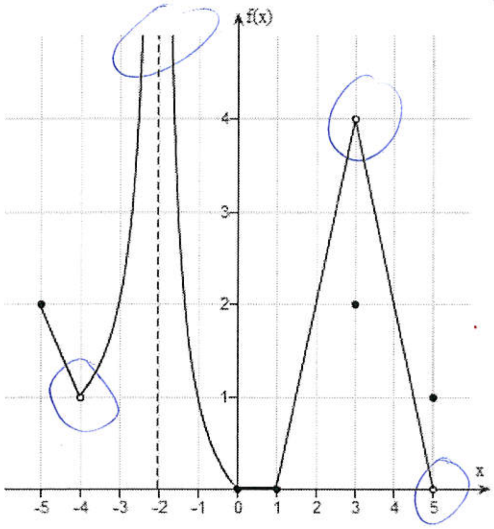
Because $x=-2$ & $x=-7$ make the denominator 0 but not the numerator, the vertical asymptotes are

$\boxed{\text{V.a.: } x = -2 \text{ \& } x = -7}$

Because degree$(x^2-4x-21)$ = degree$(x^2+9x+14)$, There ~~is~~ no oblique ~~asymptotes~~ and the ~~no~~ horizontal asymptotes is the ratio of the leading coefficients

$\boxed{\text{h.a.: } y = 1}$

5.) Determine the ~~points~~ x values at which the function f has discontinuities. At each point, state the first of the three conditions for continuity that it has failed.



The function f has discontinuities at the x-values $\boxed{x=-4,-2,3\ \&\ 5}$

$\boxed{\text{At } x=-4}$ condition 1 for continuity is failed since $\boxed{f(-4)\text{ DNE}}$

At $x=-2$, condition 1 for continuity is failed since $\boxed{f(-2)\text{ DNE}}$

At $x=3$, condition 3 for continuity is failed since $\boxed{\lim_{x\to 3} f(x) \neq f(3)}$

At $x=5$, condition 2 for continuity fails since $\boxed{\lim_{x\to 5} f(x)\text{ DNE}}$

6.) Consider the function $f(x) = \sqrt{2x-1}$.

a. Find $f'(x)$ by using the limit definition.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2(x+h)-1}-\sqrt{2x-1}}{h} = \lim_{h\to 0} \frac{\sqrt{2x+2h-1}-\sqrt{2x-1}}{h}\left(\frac{\sqrt{2x+2h-1}+\sqrt{2x-1}}{\sqrt{2x+2h-1}+\sqrt{2x-1}}\right)$$

$$= \lim_{h\to 0} \frac{(\sqrt{2x+2h-1})^2 + \sqrt{2x+2h-1}\sqrt{2x-1} - \sqrt{2x+2h-1}\sqrt{2x-1} - (\sqrt{2x-1})^2}{h(\sqrt{2x+2h-1}+\sqrt{2x-1})}$$

$$= \lim_{h\to 0}\left(\frac{2x+2h-1-2x+1}{h(\sqrt{2x+2h-1}+\sqrt{2x-1})}\right) = \lim_{h\to 0} \frac{2h}{h(\sqrt{2x+2h-1}+\sqrt{2x-1})} = \lim_{h\to 0} \frac{2}{\sqrt{2x+2h-1}+\sqrt{2x-1}}$$

$$= \frac{2}{\sqrt{2x+2\cdot 0-1}+\sqrt{2x-1}} = \frac{2}{\sqrt{2x-1}+\sqrt{2x-1}} = \frac{2}{2\sqrt{2x-1}} = \boxed{\frac{1}{\sqrt{2x-1}}}$$

b. Using the derivative you found it part a, what is the slope of the function at x = 7.

$$f'(7) = \frac{1}{\sqrt{2\cdot 7-1}} = \frac{1}{\sqrt{14-1}} = \boxed{\frac{1}{\sqrt{13}}}$$