Answer Key

Math 180
Worksheet 5
Section 3.3-3.4

1.) For each of the following functions, (i) use Product Rule to find the derivative, and then (ii) find the derivative by foiling first.

a. $y = 4t^3(-3t^2 + 14)$

(i) $y' = \frac{d}{dt}(4t^3)(-3t^2+14) + 4t^3\frac{d}{dt}(-3t^2+14)$

$= 12t^2(-3t^2+14) + 4t^3(-6t)$

$= -36t^4 + 168t^2 - 24t^4$

$= \boxed{-60t^4 + 168t^2}$ $= \boxed{12t^4 + 168t^2}$

or $= \boxed{12t^2(-5t^2+14)}$ or $= \boxed{12t^2(t^2+14)}$

(ii) $y = -12t^5 + 56t^3$

$y' = \boxed{-60t^4 + 168t^2}$

or

$y' = \boxed{12t^2(-5t^2+14)}$

b. $h(z) = (z^2 + 3)(4z - 5)$

(i) $h'(z) = \frac{d}{dz}(z^2+3)(4z-5) + (z^2+3)\frac{d}{dz}(4z-5)$

$= 2z(4z-5) + (z^2+3)(4)$

$= 8z^2 - 10z + 4z^2 + 12$

$= \boxed{12z^2 - 10z + 12}$

(ii) $h(z) = 4z^3 - 5z^2 + 12z - 15$

$\boxed{h'(z) = 12z^2 - 10z + 12}$

2.) Use Quotient Rule to find the derivative for the following functions.

a. $y = \frac{2x}{x-3}$

$y' = \frac{\frac{d}{dx}(2x)(x-3) - (2x)\frac{d}{dx}(x-3)}{(x-3)^2} = \frac{2(x-3) - 2x(1)}{(x-3)^2}$

$y' = \frac{2x - 6 - 2x}{(x-3)^2}$

$\boxed{y' = \frac{-6}{(x-3)^2}}$

b. $f(x) = (2\sqrt{x} - 1)(3x + 1)^{-1} = \frac{2\sqrt{x} - 1}{3x+1}$

$f'(x) = \frac{\frac{d}{dx}(2\sqrt{x}-1)(3x+1) + (2\sqrt{x}-1)\frac{d}{dx}(3x+1)}{(3x+1)^2} = \frac{2\cdot\frac{1}{2}x^{-1/2}(3x+1) + (2\sqrt{x}-1)3}{(3x+1)^2}$

$= \frac{3x^{1/2} + x^{-1/2} + 6\sqrt{x} - 3}{(3x+1)^2} = \boxed{\frac{9\sqrt{x} + x^{-1/2} - 3}{(3x+1)^2}}$

3.) For each of the following functions, find the first, second, and third derivatives.

a. $f(x) = \frac{1}{x} = x^{-1}$ b. $f(x) = \sqrt{x} = x^{1/2}$ c. $y = t^2 e^t$

$f'(x) = (-1)x^{-2}$

$f''(x) = 2x^{-3}$

$f'''(x) = -6x^{-4}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f''(x) = \frac{-1}{4}x^{-3/2}$

$f'''(x) = \frac{3}{8}x^{-5/2}$

$y' = 2te^t + t^2 \frac{d}{dt}(e^t)$

$y' = 2te^t + t^2e^t = e^t(t^2+2t)$

$y'' = \frac{d}{dt}(e^t)(t^2+2t) + e^t\frac{d}{dt}(t^2+2t)$

$y'' = e^t(t^2+2t) + e^t(2t+2)$

$y'' = e^t(t^2+4t+2)$

$y''' = \frac{d}{dt}(e^t)(t^2+4t+2) + e^t\frac{d}{dt}(t^2+4t+2)$

$y''' = e^t(t^2+4t+2) + e^t(2t+4)$

$y''' = e^t(t^2+6t+6)$

4.) Find the derivatives for the following functions.

a. $y = \sin x + \tan x$ b. $f(x) = 4\csc x$

$y' = \cos(x) + \sec^2(x)$

$f'(x) = 4(-\csc(x)\cot(x)) = -4\csc(x)\cot(x)$

5.) Use Product or Quotient Rule to find the derivatives for the following.

a. $f(x) = \cos x \tan x$ b. $g(x) = x \sin x$ c. $f(\theta) = \frac{1-\tan\theta}{1+\tan\theta}$

a. $f'(x) = \frac{d}{dx}(\cos(x))\tan(x) + \cos(x)\frac{d}{dx}(\tan(x))$

$= -\sin(x)\tan(x) + \cos(x)\sec^2(x)$

$= -\sin(x)\frac{\sin(x)}{\cos(x)} + \cos(x)\frac{1}{\cos^2(x)}$

$= \frac{-\sin^2(x)+1}{\cos(x)} = \frac{\cos^2(x)}{\cos(x)} = \cos(x)$

b. $g'(x) = \frac{d}{dx}(x)\sin(x) + x\frac{d}{dx}(\sin(x))$

$g'(x) = \sin(x) + x\cos(x)$