Answer Key

Math 180
Worksheet 6
Sections 3.5-3.7

1.) Suppose a stone is thrown vertically upward from the edge of a cliff with an initial velocity of ~~60~~ 64 ft/s from a height of ~~34~~ 32 feet above the ground. The height s (in ft) of the stone above the ground in t seconds after it is thrown is given by $s(t) = -16t^2 + 64t + 32$.

a. Determine the velocity v of the stone after t seconds.

$s'(t) = v(t) = \boxed{-32t + 64 \ \frac{ft}{sec}}$

b. When does the stone reach its highest point?

$s'(t) = v(t) = 0 \Rightarrow -32t + 64 = 0 \Rightarrow \boxed{t = 2 \text{ sec}}$

c. What is the height of the stone at its highest point?

$s(2) = -16(2)^2 + 64(2) + 32 = -64 + 128 + 32 = \boxed{96 \text{ ft}}$

d. When does the stone strike the ground?

$s(t) = 0 = -16t^2 + 64t + 32 \Rightarrow 0 = t^2 - 4t - 2$

$\Rightarrow t = \frac{4 \pm \sqrt{16 - 4(1)(-2)}}{2(1)} = \frac{4 \pm \sqrt{16+8}}{2} = 2 \pm \frac{1}{2}\sqrt{24} = 2 \pm \frac{1}{2}\sqrt{6}\sqrt{4} = 2 \pm \sqrt{6}$

$\Rightarrow \boxed{t = 2 + \sqrt{6}}$

e. With what velocity does the stone strike the ground?

$v(2+\sqrt{6}) = -32(2+\sqrt{6}) + 64 = -64 + 32\sqrt{6} + 64$

$= \boxed{32\sqrt{6} \ \frac{ft}{s}}$

f. Determine the acceleration a of the stone after t seconds.

$a(t) = v'(t) = s''(t) = -32 \ \frac{ft}{s^2}$

g. On what intervals is the speed increasing?

$sp(t) = |v(t)| = \begin{cases} -32t + 64 & \text{if } t \le 2 \\ 32t - 64 & \text{if } t > 2 \end{cases}$

$sp'(t) = \begin{cases} -32 & \text{if } t \le 2 \\ 32 & \text{if } t > 2 \end{cases}$

the speed is increasing on the interval $\boxed{(2, 2+\sqrt{6})}$

2.) Use Chain Rule to calculate the derivatives of the following functions.

a. $f(x) = (3x + 2)^{14}$

$$f'(x) = 14(3x+2)^{13} \cdot 3$$

b. $\ln(3x + 8)$

$$y' = \frac{1}{3x+8} \cdot 3$$

c. $h(t) = e^{t^3+6t^2-10t-3}$

$$h'(t) = \left(e^{t^3+6t^2-10t-3}\right)(3t^2+12t-10)$$

d. $y = \sin(4\cos\theta)$

$$y' = \cos(4\cos\theta) \cdot (-4\sin\theta)$$

3.) Find the derivatives of the following functions by making use of the Chain Rule more than once.

a. $y = \sqrt{(2x-7)^3 + 5x} = ((2x-7)^3+5x)^{1/2}$

$$y' = \frac{1}{2}((2x-7)^3+5x)^{-1/2} \cdot \frac{d}{dx}((2x-7)^5+5x)$$

$$= \boxed{\frac{1}{2}((2x-7)^3+5x)^{-1/2}(5(2x-7)^4 \cdot 2+5)}$$

b. $f(t) = \cos(2^{5t+3})$

$$f'(t) = \sin(2^{5t+3}) \cdot \frac{d}{dt}(2^{5t+3})$$

$$= \boxed{\sin(2^{5t+3})\, 2^{5t+3} \ln(2) \cdot 5}$$

4.) Combine Chain Rule with Product or Quotient Rule to find the derivative of the following.

a. $f(x) = (\frac{e^x}{x+1})^8$ $\cancel{= 8\left(\frac{e^x}{x+1}\right)}$

$$f'(x) = 8\left(\frac{e^x}{x+1}\right)^7 \cdot \frac{d}{dx}\left(\frac{e^x}{x+1}\right) = 8\left(\frac{e^x}{x+1}\right)^7\left(\frac{\cancel{e^x \frac{d}{dx}(x+1)}\ \frac{d}{dx}(e^x)(x+1) + e^x\left(\frac{d}{dx}(x+1)\right)}{(x+1)^2}\right)$$

$$= \boxed{8\left(\frac{e^x}{x+1}\right)^7 \frac{e^x(x+1)+e^x(1)}{(x+1)^2} = 8\frac{(e^x)^8 x}{(x+1)^9}}$$

b. $g(x) = \tan(x \cdot 2^x)$

$g'(x) = \sec^2(x2^x) \cdot \frac{d}{dx}(x2^x)$

$= \sec^2(x2^x)\left(\frac{d}{dx}(x)2^x + x\frac{d}{dx}(2^x)\right)$

$\boxed{= \sec^2(x2^x)(2^x + x2^x\ln(2)) = \sec^2(x2^x)2^x(1+x\ln(2))}$

5.) Use implicit differentiation to find $\frac{dy}{dx}$.

a. $y^2 = 4x$

$2yy' = 4$

$y' = \frac{4}{2y} = \boxed{\frac{2}{y} = y'}$

b. $x^3 = \frac{x+y}{x-y}$

$x^3(x-y) = x+y$

$x^4 - x^3y = x+y$

$4x^3 - \left(\frac{d}{dx}(x^3)y + x^3\frac{d}{dx}(y)\right) = 1 + y'$

$4x^3 - (3x^2y + x^3y') = 1 + y'$

$4x^3 - 3x^2y + x^3y' = 1 + y'$

$4x^3 - 3x^2y - 1 = y' - x^3y' = y'(1-x^3)$

$\boxed{y' = \frac{4x^3 - 3x^2y - 1}{1 - x^3}}$

c. $\cos y^2 + x = e^y$

6.) Use implicit differentiation to find $\frac{dy}{dx}$.

a. $xy^{5/2} + x^{3/2}y = 12$

$\left(\frac{d}{dx}(x)y^{5/2} + x\frac{d}{dx}(y^{5/2})\right) + \left(\frac{d}{dx}(x^{3/2})y + x^{3/2}\frac{d}{dx}(y)\right) = 0$

$y^{5/2} + x\frac{5}{2}y^{3/2}y' + \frac{3}{2}x^{1/2}y + x^{3/2}y' = 0$

$\left(\frac{5}{2}xy^{3/2} + x^{3/2}\right)y' = -\left(y^{5/2} + \frac{3}{2}x^{1/2}y\right) \rightarrow \boxed{y' = \frac{-\left(y^{5/2} + \frac{3}{2}x^{1/2}y\right)}{\frac{5}{2}xy^{3/2} + x^{3/2}}}$

b. $\sin x \cos y = \sin x + \cos y$

$\frac{d}{dx}(\sin(x))\cos(y) + \sin(x)\frac{d}{dx}(\cos(y)) = \cos(x) - \sin(y)y'$

$\cos(x)\cos(y) - \sin(x)\sin(y)y' = \cos(x) - \sin(y)y'$

$\cos(x)\cos(y) - \cos(x) = \sin(x)\sin(y)y' - \sin(y)y'$

$\boxed{\frac{\cos(x)(\cos(y)-1)}{\sin(y)(\sin(x)-1)} = y'}$