Answer Key

Math 180
Worksheet 7
Section 3.8-3.9

1.) Find the derivatives for the following.

a. $y = 5^x$

$y' = 5^x \ln(5)$

b. $f(x) = \log_2 x = \frac{\ln(x)}{\ln(2)} = \frac{1}{\ln(2)} \ln(x)$

$f'(x) = \frac{1}{\ln(2)} \cdot \frac{1}{x} = \boxed{\frac{1}{x\ln(2)}}$

2.) Use Product or Quotient Rule to find the derivatives for the following.

a. $y = 3^x \log_6 x$

b. $f(x) = \frac{e^{-x} \log x}{2^x}$

(a) $y' = 3^x \ln(3) \cdot \log_6(x) + 3^x \frac{1}{\ln(6)x}$

(b) $f'(x) = \frac{\frac{d}{dx}(e^{-x}\log(x))2^x + e^{-x}\log(x)\frac{d}{dx}(2^x)}{(2^x)^2} = \frac{\left(\frac{d}{dx}(e^{-x})\log(x) + e^{-x}\frac{d}{dx}(\log(x))\right)2^x + e^{-x}\log(x)2^x\ln(2)}{2^{2x}}$

$= \boxed{\frac{-e^{-x}\log(x) + e^{-x}\frac{1}{\ln(10)x}2^x + e^{-x}\log(x)2^x\ln(2)}{2^{2x}}}$

or $= \boxed{\frac{e^{-x}(-\log(x) + 2^x(\ln(10)x)^{-1} + \log(x)2^x\ln(2))}{4^x}}$

3.) Use properties of logarithms to re-write the equation before computing the derivative. $f(x) = (x^2 + 5)^{sinx}$

$\ln(f(x)) = \ln((x^2+5)^{\sin(x)}) = \sin(x)\ln(x^2+5)$

$\frac{f'(x)}{f(x)} = \cos(x)\ln(x^2+5) + \sin(x)\frac{1}{x^2+5} \cdot 2x$

$f'(x) = f(x)\left(\cos(x)\ln(x^2+5) + \sin(x)\frac{2x}{x^2+5}\right)$

$\boxed{f'(x) = (x^2+5)^{\sin(x)}\left(\cos(x)(x^2+5) + \sin(x)\frac{2x}{x^2+5}\right)}$

4.) Find the derivatives of the following functions: (Any variable in the equation that is not the variable of differentiation is a function of that variable)

$y' = \frac{dy}{dx}$

a. $f(x) = cot^{-1}(y^2 - x)$

$$\boxed{f'(x) = \frac{-1}{1+(y^2-x)^2} \cdot (2yy'-1)}$$

b. $g(y) = csc^{-1}(2y)$

$$\boxed{g'(y) = \frac{1}{|2y|\sqrt{4y^2-1}} \cdot 2}$$

c. $h(w) = sin^{-1}(w)tan^{-1}(x)$

$$h'(w) = \frac{d}{dw}(sin^{-1}(w))\, tan^{-1}(x) + sin^{-1}(w) \frac{d}{dw}(tan^{-1}(x))$$

$$= \boxed{\frac{1}{\sqrt{1-w^2}} tan^{-1}(x) + sin^{-1}(w) \frac{1}{1+x^2} \frac{dx}{dw}}$$