Math 180
Worksheet 8
Section 3.10, 4.1

1.) Consider the equation $5y^2 = 6x - xy$, where x and y are both functions of time. If $\frac{dx}{dt} = 3$ when $x = 2$, find dy/dt when $x = 2$.

$10yy' = 6x' - x'y - xy'$

$10yy' + xy' = 6x' - x'y$

$\frac{dy}{dt} = \frac{(6-y)\frac{dx}{dt}}{10y + x}$

$x = 2 \Rightarrow 5y^2 = 12 - 2y$

$5y^2 + 2y - 12 = 0$

$y = \frac{-2 \pm \sqrt{4 - 4(5)(12)}}{10} = \frac{-1 \pm \sqrt{61}}{5}$

$x = 2 \Rightarrow y = \frac{-1 \pm \sqrt{61}}{5}$ & $\frac{dx}{dt} = 3$

so $\boxed{\frac{dy}{dt} = \frac{\left(6 - \frac{-1 \pm \sqrt{61}}{5}\right) 3}{10\left(\frac{-1 \pm \sqrt{61}}{5}\right) + 2}}$ when $x = 2$

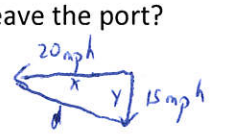
2.) A spherical balloon is inflated and its volume increases at a rate of 15 in^3 / min. What is the rate of change of its radius when the radius is 10 in?

V = volume, r = radius $\quad V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 15 \frac{in^3}{min}$, $r = 10\,in \Rightarrow 15 \frac{in^3}{min} = 4\pi (10\,in)^2 \frac{dr}{dt}$

$\Rightarrow \boxed{\frac{dr}{dt} = \frac{3}{80\pi} \frac{in}{min}}$

3.) Two boats leave a port at the same time; one travels west at 20 mi/hr and the other travels south at 15 mi/hr. At what rate is the distance between them changing 30 minutes after they leave the port?



$d^2 = x^2 + y^2$

$2dd' = 2xx' + 2yy'$

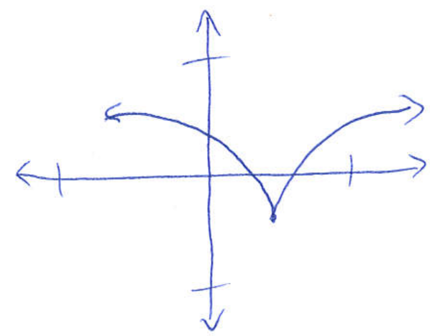
$30\,min = t \Rightarrow x = 10\,mi,\ y = 7.5\,mi \Rightarrow d = \sqrt{10^2 + 7.5^2} = 12.5\,mi$

$d' = \frac{2xx' + 2yy'}{2d} = \frac{xx' + yy'}{d}$

$d' = \frac{10\,mi \cdot 20\frac{mi}{hr} + 7.5\,mi \cdot 15\frac{mi}{hr}}{12.5\,mi} = \frac{2(10^2 + 7.5^2)}{12.5} \frac{mi}{hr} = \frac{2 \cdot 12.5^2}{12.5} \frac{mi}{hr}$

$\Rightarrow \boxed{\frac{dr}{dt} = 25\,mph}$

4.) Sketch the graph of a function that has a local minimum value at a point c where $f'(c)$ is undefined.



5.) Find the critical points of the following function, and then find the absolute max and absolute min on the interval given.

$f(x) = x^{2/3}$; $[-8,8]$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

$f'(x)$ is undefined at $x=0$

$f'(x) \neq 0$ so $x = 0$ is the only crit. pt.

$f(-8) = (-8)^{2/3} = 4$

$f(0) = 0^{2/3} = 0$

$f(8) = 8^{2/3} = 4$

So the absolute max on $[-8,8]$ is 4 & it occurs at $x=-8$ & $x=8$

The absolute min on $[-8,8]$ is 0 & occurs at $x=0$