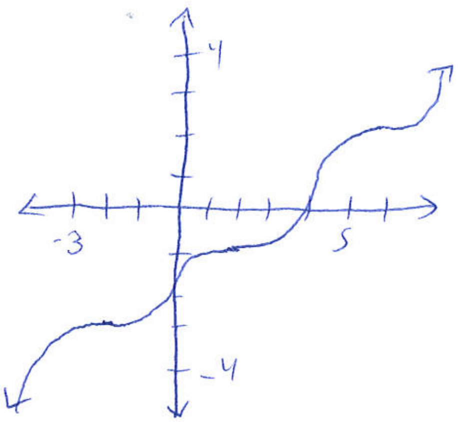
Math 180
Worksheet 9
Section 4.2

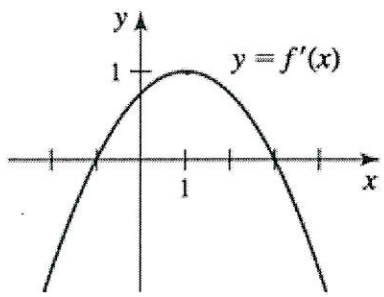
1.) Sketch a graph of a function that is continuous for all reals and has the following properties. Use a sign graph to summarize information about the function.

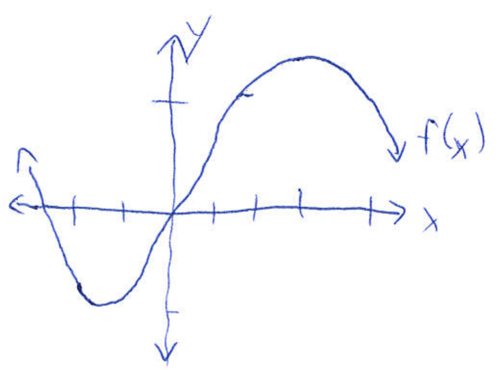
$$f'(-2) = f'(2) = f'(6) = 0\,;\, f'(x) \geq 0 \text{ on } (-\infty, \infty)$$



x	$(-\infty,-2)$	$(-2,2)$	$(2,6)$	$(6,\infty)$
$f'(x)$	+	+	+	+

2.) The following figure gives the graph of the *derivative* of a continuous function f that passes through the origin. Sketch a possible graph of f on the same set of axes.

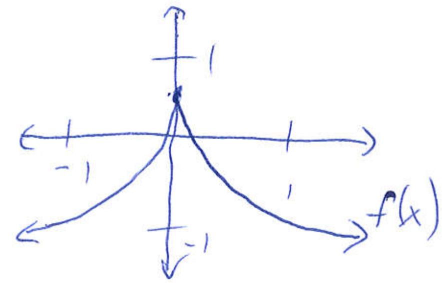




3.) Sketch a graph of a function f that is continuous on $(-\infty, \infty)$ and has the following properties:

$$f'(x) > 0 \text{ and } f''(x) > 0 \text{ on } (-\infty, 0)\ ;\ f'(x) < 0 \text{ and } f''(x) < 0 \text{ on } (0, \infty)$$

x	$(-\infty, 0)$	$(0, \infty)$
$f'(x)$	+	−
inc/dec	inc	dec
$f''(x)$	+	−
concave up/down	up	down
	◡	(



4.) Locate the critical points of the following functions. Then use the second derivative test to determine (if possible) whether they correspond to local maxima or local minima.

a. $f(x) = -x^3 - 6$ b. $f(x) = \frac{x^4}{2-12x^2}$

c. $f(x) = x^2 e^{-x}$ d. $f(x) = tan^{-1}(x)$

(a) $f'(x) = -3x^2$, f' is defined everywhere, $f'(x) = 0 \Rightarrow -3x^2 = 0 \Rightarrow x = 0$ so $x = 0$ is the crit. pt.

$f''(x) = -6x$, $f''(0) = 0$ so the 2nd derivative test is inconclusive

x	$(-\infty, 0)$	$(0, \infty)$
$f'(x)$	−	+

so $x = 0$ is where a local min. occurs

(b) $f'(x) = 2xe^{-x} - x^2e^{-x} = e^{-x}(2x - x^2)$, f' is defined everywhere, $f'(x) = 0 \Rightarrow e^{-x}(2x - x^2) = 0 \Rightarrow 2x - x^2 = 0$ since $e^{-x} \neq 0$

$\Rightarrow x(2 - x) = 0 \Rightarrow x = 2$ or 0 are the crit. pt.s

$f''(x) = (2 - 2x)e^{-x} - (2x - x^2)e^{-x} = (2 - 4x + x^2)e^{-x}$

$f''(0) = 2$ so 0 is where a local min. occurs, $f''(2) = -2e^{-2}$ so a local max occurs there

(d) $f'(x) = \frac{1}{1+x^2}$, $f'(x) = 0 \Rightarrow \frac{1}{1+x^2} = 0 \Rightarrow 1 \neq 0$ ~~contr~~ impossible

$f'(x)$ is defined everywhere

(b) $f'(x) = \frac{2x^3(1-3x^2)}{(1-6x^2)^2}$, $f'(x)$ is undefined if $1 - 6x^2 = 0 \Rightarrow x = \frac{\pm 1}{\sqrt{6}}$

$f'(x) = 0 \Rightarrow 2x^3(1 - 3x^2) = 0 \Rightarrow x = 0$ or $\frac{\pm 1}{\sqrt{3}}$ So the crit. pts are $0, \frac{\pm 1}{\sqrt{6}}, \frac{\pm 1}{\sqrt{3}}$

$f''(x) = \frac{6x^2(6x^4 - 3x^2 + 1)}{(6x^2-1)^3}$, $f''(0) = 0$, $f''\left(\frac{\pm 1}{\sqrt{6}}\right)$ undefined, $f''\left(\frac{\pm 1}{\sqrt{3}}\right) = \frac{-4}{3}$ so ~~0 has a~~ the 2nd der. test is inconclusive about $x = 0$ & $\frac{\pm 1}{\sqrt{6}}$ but says a local max occurs at $x = \frac{\pm 1}{\sqrt{3}}$