Summer 2014
Review Day 7

Change of Variables

1. Let $\Phi(u,v) = (u^2, v)$ is Φ one to one? If not determine a domain on which Φ is one to one. Find the image under Φ of:

(a) The u and v axes

(b) The rectangle $[-1,1] \times [-1,1]$

(c) The line segment joining $(0,0)$ and $(1,1)$

(d) The triangle with vertices $(0,0), (0,1)$ and $(1,1)$

2. Let $\Phi(u,v) = (2u+v, 5u+3v)$ be a map from the $uv-$plane to the xy-plane.

(a) Show that the image of the horizontal line $v = c$ is the line $y = \frac{5}{2}x + \frac{1}{2}c$. What is the image of the line $u = c$.

(b)Describe the image of the line $v = 4u$ under Φ.

(c) Show that the inverse of Φ is $\Phi^{-1}(x,y) = (3x - y, -5x + 2y)$

3. Calculate the Jacobian

(a) $\Phi(u,v) = (3u + 4v, u - 2v)$

(b) $\Phi(u,v) = (ue^v, ve^{3u})$

(c) $\Phi(r,\theta) = (r\cos\theta, r\sin\theta)$

4. Find a linear mapping Φ that maps $[0,1] \times [0,1]$ to the parallelogram in the xy-plane spanned by the vectors $\langle 2,3\rangle$ and $\langle 4,1\rangle$.